il grado del numeratore è minore del grado del denominatore (sicché la parte intera è nulla), e poi quelli aventi parte intera non nulla, nei quali occorre preventivamente effettuare la divisione del numeratore per il denominatore allo scopo di determinare la parte intera.

Infine riporteremo alcuni integrali in cui conviene prima effettuare una sostituzione.

9. Integrali di fratti semplici, cioè di funzioni razionali del tipo:

$$(*) \qquad \int \frac{a}{(x-x_0)^n}\,dx \quad , \quad \int \frac{ax+b}{(x^2+px+q)^n}\,dx$$

con a, b, p, q numeri reali, $n \in N$ e $p^2-4q<0$ [cfr. (2) pag. 468].

Notiamo che anche gli integrali:

$$\int \frac{\alpha}{(\lambda x+\mu)^n}\,dx \quad , \quad \int \frac{\alpha x+\beta}{(\lambda x^2+\mu x+\nu)^n}\,dx \qquad (\mu^2-4\lambda\nu<0)$$

sono del tipo $(*)$, in quanto si ha:

$$\frac{\alpha}{(\lambda x+\mu)^n} = \frac{\frac{\alpha}{\lambda^n}}{\left(x+\frac{\mu}{\lambda}\right)^n} \quad , \quad \frac{\alpha x+\beta}{(\lambda x^2+\mu x+\nu)^n} = \frac{\frac{\alpha}{\lambda^n}x+\frac{\beta}{\lambda^n}}{\left(x^2+\frac{\mu}{\lambda}x+\frac{\nu}{\lambda}\right)^n}$$

- $\int \frac{dx}{(x-2)^3} = -\frac{1}{2}\frac{1}{(x-2)^2}$, $\int \frac{dx}{(x+3)^4} = -\frac{1}{3}\frac{1}{(x+3)^3}$, $\int \frac{dx}{(x+1)^5} = -\frac{1}{4}\frac{1}{(x+1)^4}$,
- $\int \frac{dx}{(2x+3)^4} = -\frac{1}{6}\frac{1}{(2x+3)^3}$, $\int \frac{dx}{(3x-1)^5} = -\frac{1}{12}\frac{1}{(3x-1)^4}$, $\int \frac{dx}{(5x+2)^4} = -\frac{1}{15}\frac{1}{(5x+2)^3}$,
- $\int \frac{2x+3}{x^2+2x+2}\,dx = \log(x^2+2x+2)+\operatorname{arctg}(x+1)$, $\int \frac{3x-1}{x^2+x+1}\,dx = \frac{3}{2}\log(x^2+x+1)-\frac{5}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}$
- $\int \frac{3x+4}{x^2+x+2}\,dx = \frac{3}{2}\log(x^2+x+2)+\frac{5}{\sqrt{7}}\operatorname{arctg}\frac{2x+1}{\sqrt{7}}$, $\int \frac{5x+1}{x^2+2x+3}\,dx = \frac{5}{2}\log(x^2+2x+3)-\frac{4}{\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}}$,
- $\int \frac{3x-2}{x^2+3x+7}\,dx = \frac{3}{2}\log(x^2+3x+7)-\frac{13}{\sqrt{19}}\operatorname{arctg}\frac{2x+3}{\sqrt{19}}$, $\int \frac{2x+5}{x^2+x+3}\,dx = \log(x^2+x+3)+\frac{8}{\sqrt{11}}\operatorname{arctg}\frac{2x+1}{\sqrt{11}}$,
- $\int \frac{3x+2}{2x^2+x+1}\,dx = \frac{3}{4}\log(2x^2+x+1)+\frac{5}{2\sqrt{7}}\operatorname{arctg}\frac{4x+1}{\sqrt{7}}$, $\int \frac{3x-1}{2x^2+2x+5}\,dx = \frac{3}{4}\log(2x^2+2x+5)-\frac{5}{6}\operatorname{arctg}\frac{2x+1}{3}$,
- $\int \frac{5x-1}{3x^2+x+1}\,dx = \frac{5}{6}\log(3x^2+x+1)-\frac{\sqrt{11}}{3}\operatorname{arctg}\frac{6x+1}{\sqrt{11}}$, $\int \frac{2x+3}{5x^2+x+1}\,dx = \frac{1}{5}\log(5x^2+x+1)+\frac{28}{5\sqrt{19}}\operatorname{arctg}\frac{10x+1}{\sqrt{19}}$

$$\int \frac{dx}{(1+x^2)^2} = \frac{1}{2}\left(\operatorname{arctg} x + \frac{x}{1+x^2}\right)\ ^{(1)}\ , \quad \int \frac{dx}{(1+x^2)^3} = \frac{3}{8}\operatorname{arctg} x + \frac{3x}{8(1+x^2)} + \frac{x}{4(1+x^2)^2},$$

$$\int \frac{dx}{(1+x^2)^4} = \frac{5}{16}\operatorname{arctg} x + \frac{5x}{16(1+x^2)} + \frac{5x}{24(1+x^2)^2} + \frac{x}{6(1+x^2)^3},$$

$$\int \frac{dx}{(x^2+x+1)^2} = \frac{1}{3}\left(\frac{4}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}} + \frac{2x+1}{x^2+x+1}\right), \quad \int \frac{dx}{(x^2+x+2)^2} = \frac{1}{7}\left(\frac{4}{\sqrt{7}}\operatorname{arctg}\frac{2x+1}{\sqrt{7}} + \frac{2x+1}{x^2+x+2}\right),$$

$$\int \frac{dx}{(x^2+2x+5)^2} = \frac{1}{16}\left(\operatorname{arctg}\frac{x+1}{2} + \frac{2x+2}{x^2+2x+5}\right), \quad \int \frac{dx}{(x^2+3x+5)^2} = \frac{1}{11}\left(\frac{4}{\sqrt{11}}\operatorname{arctg}\frac{2x+3}{\sqrt{11}} + \frac{2x+3}{x^2+3x+5}\right),$$

$$\int \frac{3x+2}{(1+x^2)^2}\,dx = \operatorname{arctg} x + \frac{1}{2}\,\frac{2x-3}{1+x^2}, \quad \int \frac{5x+1}{(1+x^2)^3}\,dx = \frac{3}{8}\operatorname{arctg} x + \frac{3x}{8(1+x^2)} + \frac{x-5}{4(1+x^2)^2},$$

$$\int \frac{3x+1}{(1+x^2)^4}\,dx = \frac{5}{16}\operatorname{arctg} x + \frac{5x}{16(1+x^2)} + \frac{5x}{24(1+x^2)^2} + \frac{x-3}{6(1+x^2)^3}$$

$$\int \frac{3x+7}{(x^2+2x+5)^2}\,dx = \frac{1}{4}\operatorname{arctg}\frac{x+1}{2} + \frac{x-2}{2(x^2+2x+5)}, \quad \int \frac{x-1}{(x^2+x+1)^2}\,dx = -\frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}} - \frac{x+1}{x^2+x+1}$$

$$\int \frac{15x+7}{(5x^2+2x+1)^2}\,dx = \frac{5}{4}\operatorname{arctg}\frac{5x+1}{2} + \frac{1}{2}\,\frac{5x-2}{5x^2+2x+1}.$$

10. Integrali di funzioni razionali che non sono fratti semplici [n. 9 pag. 468].

1 $\displaystyle\int \frac{x+1}{x^2-5x+6}\,dx = -3\int\frac{dx}{x-2} + 4\int\frac{dx}{x-3} = -3\log|x-2| + 4\log|x-3|$

2 $\displaystyle\int \frac{3x+4}{x^2+3x+2}\,dx = \int\frac{dx}{x+1} + 2\int\frac{dx}{x+2} = \log|x+1| + 2\log|x+2|$

3 $\displaystyle\int \frac{3x+1}{x^2+2x-3}\,dx = \int\frac{dx}{x-1} + 2\int\frac{dx}{x+3} = \log|x-1| + 2\log|x+3|$

4 A $\displaystyle\int \frac{x-7}{x^2+4x-5}\,dx = -\int\frac{dx}{x-1} + 2\int\frac{dx}{x+5} = -\log|x-1| + 2\log|x+5|$

5 A $\displaystyle\int \frac{dx}{x^2-2x-3} = -\frac{1}{4}\int\frac{dx}{x+1} + \frac{1}{4}\int\frac{dx}{x-3} = \frac{1}{4}\log\left|\frac{x-3}{x+1}\right|$

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(1) Cfr. n. 6 pag. 459

6 $\displaystyle\int\frac{dx}{x^2-3x+2} = -\int\frac{dx}{x-1} + \int\frac{dx}{x-2} = \log\left|\frac{x-2}{x-1}\right|$

7 $\displaystyle\int\frac{dx}{x^2-x-6} = -\frac{1}{5}\int\frac{dx}{x+2} + \frac{1}{5}\int\frac{dx}{x-3} = \frac{1}{5}\log\left|\frac{x-3}{x+2}\right|$

8 $\displaystyle\int\frac{dx}{x^2+x-2} = \frac{1}{3}\int\frac{dx}{x-1} - \frac{1}{3}\int\frac{dx}{x+2} = \frac{1}{3}\log\left|\frac{x-1}{x+2}\right|$

9 $\displaystyle\int\frac{dx}{x^2+5x+6} = \int\frac{dx}{x+2} - \int\frac{dx}{x+3} = \log\left|\frac{x+2}{x+3}\right|$

10 A $\displaystyle\int\frac{dx}{x^2-3x} = -\frac{1}{3}\int\frac{dx}{x} + \frac{1}{3}\int\frac{dx}{x-3} = \frac{1}{3}\log\left|\frac{x-3}{x}\right|$, A $\displaystyle\int\frac{dx}{x^2+x} = \int\frac{dx}{x} - \int\frac{dx}{x+1} = \log\left|\frac{x}{x+1}\right|$

11 A $\displaystyle\int\frac{dx}{x^2-4} = -\frac{1}{4}\int\frac{dx}{x+2} + \frac{1}{4}\int\frac{dx}{x-2} = \frac{1}{4}\log\left|\frac{x-2}{x+2}\right|$

12 $\displaystyle\int\frac{2x-1}{3x^2+7x+2}\,dx = \int\frac{dx}{x+2} - \int\frac{dx}{3x+1} = \log|x+2| - \frac{1}{3}\log|3x+1|$

13 $\displaystyle\int\frac{x+1}{6x^2+7x+2}\,dx = \int\frac{dx}{2x+1} - \int\frac{dx}{3x+2} = \frac{1}{2}\log|2x+1| - \frac{1}{3}\log|3x+2|$

14 $\displaystyle\int\frac{x\,dx}{2x^2-3x+1} = \int\frac{dx}{x-1} - \int\frac{dx}{2x-1} = \log|x-1| - \frac{1}{2}\log|2x-1|$

15 $\displaystyle\int\frac{dx}{4x^2-4x-3} = \frac{1}{4}\int\frac{dx}{2x-3} - \frac{1}{4}\int\frac{dx}{2x+1} = \frac{1}{8}\log\left|\frac{2x-3}{2x+1}\right|$

16 $\displaystyle\int\frac{dx}{4x^2-8x-5} = \frac{1}{6}\int\frac{dx}{2x-5} - \frac{1}{6}\int\frac{dx}{2x+1} = \frac{1}{12}\log\left|\frac{2x-5}{2x+1}\right|$

17 $\displaystyle\int\frac{dx}{9x^2+3x-2} = \frac{1}{3}\int\frac{dx}{3x-1} - \frac{1}{3}\int\frac{dx}{3x+2} = \frac{1}{9}\log\left|\frac{3x-1}{3x+2}\right|$

18 $\displaystyle\int\frac{dx}{2x^2-3x} = -\frac{1}{3}\int\frac{dx}{x} + \frac{2}{3}\int\frac{dx}{2x-3} = \frac{1}{3}\log\left|\frac{2x-3}{x}\right|$

19 ? $\displaystyle\int\frac{dx}{4x^2-9} = \frac{1}{6}\int\frac{dx}{2x-3} - \frac{1}{6}\int\frac{dx}{2x+3} = \frac{1}{12}\log\left|\frac{2x-3}{2x+3}\right|$

20 ? $\displaystyle\int\frac{dx}{9x^2-4} = \frac{1}{4}\int\frac{dx}{3x-2} - \frac{1}{4}\int\frac{dx}{3x+2} = \frac{1}{12}\log\left|\frac{3x-2}{3x+2}\right|$

21 $\int \frac{2x+11}{(x+5)^2}dx = 2\int\frac{dx}{x+5} + \int\frac{dx}{(x+5)^2} = 2\log|x+5| - \frac{1}{x+5}$

22 $\int \frac{x-5}{(x-6)^2}dx = \int\frac{dx}{x-6} + \int\frac{dx}{(x-6)^2} = \log|x-6| - \frac{1}{x-6}$

23 $\int \frac{2x-3}{(2x-5)^2}dx = \int\frac{dx}{2x-5} + 2\int\frac{dx}{(2x-5)^2} = \frac{1}{2}\log|2x-5| - \frac{1}{2x-5}$

24 $\int \frac{6x-23}{(2x-7)^2}dx = 3\int\frac{dx}{2x-7} - 2\int\frac{dx}{(2x-7)^2} = \frac{3}{2}\log|2x-7| + \frac{1}{2x-7}$

25 $\int \frac{4x+7}{(4x+5)^2}dx = \int\frac{dx}{4x+5} + 2\int\frac{dx}{(4x+5)^2} = \frac{1}{4}\log|4x+5| - \frac{1}{2}\,\frac{1}{4x+5}$

26 $\int \frac{2x^2+1}{x(x^2+1)}dx = \int\frac{dx}{x} + \int\frac{x}{x^2+1}dx = \log|x| + \frac{1}{2}\log(x^2+1)$

27 $\int \frac{(x+1)^2}{x(x^2+3)}dx = \frac{1}{3}\int\frac{dx}{x} + \frac{2}{3}\int\frac{x+3}{x^2+3}dx = \frac{1}{3}\log|x| + \frac{1}{3}\log(x^2+3) + \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}}$

28 $\int \frac{3x^2+x-2}{(x+3)(x^2+2)}dx = 2\int\frac{dx}{x+3} + \int\frac{x-2}{x^2+2}dx = \frac{1}{2}\log|x+3| + \frac{1}{2}\log(x^2+2) - \sqrt{2}\operatorname{arctg}\frac{x}{\sqrt{2}}$

$\int \frac{x^2-7x+1}{(3x+1)(x^2-x+3)}dx = \int\frac{dx}{3x+1} - 2\int\frac{dx}{x^2-x+3} = \frac{1}{3}\log|3x+1| - \frac{4}{\sqrt{11}}\operatorname{arctg}\frac{2x-1}{\sqrt{11}}$

$\int \frac{11x^2-21x+16}{(2x-1)(x^2-3x+4)}dx = 3\int\frac{dx}{2x-1} + 4\int\frac{(x-1)dx}{x^2-3x+4} = \frac{3}{2}\log|2x-1| + 2\log(x^2-3x+4) + \frac{4}{\sqrt{7}}\operatorname{arctg}\frac{2x-3}{\sqrt{7}}$

$\int \frac{4x^2+2x-21}{(4x+3)(x^2-2x+3)}dx = -4\int\frac{dx}{4x+3} + \int\frac{2x-3}{x^2-2x+3}dx = -4\log|4x+3| + \log(x^2-2x+3) - \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}}$

$\int \frac{x^2-6x+7}{(x-1)(x^2-5x+6)}dx = \int\frac{dx}{x-1} + \int\frac{dx}{x-2} - \int\frac{dx}{x-3} = \log\left|\frac{(x-1)(x-2)}{x-3}\right|$

* $\int \frac{x^2+4x-11}{x^3-2x^2-5x+6}dx = \int\frac{dx}{x-1} - \int\frac{dx}{x+2} + \int\frac{dx}{x-3} = \log\left|\frac{(x-1)(x-3)}{x+2}\right|$

$\int \frac{2x^2+x+11}{x^3+2x^2-2x+3}dx = 2\int\frac{dx}{x+3} + 3\int\frac{dx}{x^2-x+1} = 2\log|x+3| + \frac{6}{\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}}$

$\int \frac{3x^2+5x+2}{x^3+x^2+2x}dx = \int\frac{dx}{x} + 2\int\frac{x+2}{x^2+x+2}dx = \log|x| + \log(x^2+x+2) + \frac{6}{\sqrt{7}}\operatorname{arctg}\frac{2x+1}{\sqrt{7}}$

$\int \frac{x^2+4x-1}{x^3+x^2-x-1}dx = \int\frac{dx}{x-1} + 2\int\frac{dx}{(x+1)^2} = \log|x-1| - \frac{2}{x+1}$

$$\int \frac{x^3+x^2+1}{x^4+x^2}\,dx = \int\frac{dx}{x^2} + \int\frac{x}{x^2+1}\,dx = -\frac{1}{x} + \frac{1}{2}\log(x^2+1)$$

$$\int \frac{2x^3+10x^2+15x+3}{x^4+6x^3+12x^2+18x+27}\,dx = \frac{1}{2}\int\frac{dx}{x+3} - \frac{1}{2}\int\frac{dx}{(x+3)^2} + \frac{3}{2}\int\frac{x\,dx}{x^2+3} = \frac{1}{2}\log|x+3| + \frac{1}{2(x+3)} + \frac{3}{4}\log(x^2+3)$$

$$\int \frac{4x^3+3x^2+3x+7}{(2x-1)^2(x^2+2x+2)}\,dx = 3\int\frac{dx}{(2x-1)^2} + \int\frac{x+1}{x^2+2x+2}\,dx = -\frac{3}{2(2x-1)} + \frac{1}{2}\log(x^2+2x+2)$$

$$\int \frac{9x^3-13x^2+7x+5}{(3x-1)^2(x^2+3)}\,dx = 2\int\frac{dx}{(3x-1)^2} + \int\frac{x-1}{x^2+3}\,dx = -\frac{2}{3(3x-1)} + \frac{1}{2}\log(x^2+3) - \frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{x}{\sqrt{3}}$$

$$\int \frac{15x^3-9x^2+4x-1}{(3x-2)^2(x^2-2x+3)}\,dx = 2\int\frac{dx}{3x-2} + \int\frac{dx}{(3x-2)^2} + \int\frac{x+2}{x^2-2x+3}\,dx =$$

$$= \frac{2}{3}\log|3x-2| - \frac{1}{3(3x-2)} + \frac{1}{2}\log(x^2-2x+3) + \frac{3}{\sqrt{2}}\,\mathrm{arctg}\,\frac{x-1}{\sqrt{2}}$$

$$\int \frac{6x^3+11x^2-8x+4}{(2x-1)^2(x^2+x+1)}\,dx = \int\frac{dx}{2x-1} + 2\int\frac{dx}{(2x-1)^2} + \int\frac{x+3}{x^2+x+1}\,dx =$$

$$= \frac{1}{2}\log|2x-1| - \frac{1}{2x-1} + \frac{1}{2}\log(x^2+x+1) + \frac{5}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{3}}$$

$$\int \frac{x^3-16x^2-39x+74}{x^4+4x^3-7x^2-22x+24}\,dx = -\int\frac{dx}{x-1} - 2\int\frac{dx}{x-2} + \int\frac{dx}{x+3} + 3\int\frac{dx}{x+4} = \log\frac{|(x+3)(x+4)^3|}{|x-1|(x-2)^2}$$

$$\int \frac{x^3-x^2+3x-7}{x^4-4x^3+4x^2-4x+3}\,dx = \int\frac{dx}{x-1} + \int\frac{dx}{x-3} - \int\frac{x+1}{x^2+1}\,dx = \log\frac{|x^2-4x+3|}{\sqrt{x^2+1}} - \mathrm{arctg}\,x$$

$$\int \frac{x^3-3x^2+4x-3}{x^4-5x^3+9x^2-7x+2}\,dx = \int\frac{dx}{(x-1)^3} + \int\frac{dx}{x-2} = -\frac{1}{2(x-1)^2} + \log|x-2|$$

$$\int \frac{2x^3+5x^2+4x+2}{x^5+5x^4+9x^3+7x^2+2x}\,dx = \int\frac{dx}{x} - \int\frac{dx}{(x+1)^3} - \int\frac{dx}{x+2} = \log\left|\frac{x}{x+2}\right| + \frac{1}{2(x+1)^2}$$

$$\int \frac{x^3-2x^2-x+3}{(x-2)^2(x-1)^3}\,dx = \int\frac{dx}{(x-2)^2} + \int\frac{dx}{(x-1)^3} = -\frac{1}{x-2} - \frac{1}{2(x-1)^2}$$

$$\int \frac{x^4-4x^3+6x^2-5x+4}{(x-1)^4(x-3)}\,dx = -\int\frac{dx}{(x-1)^4} + \int\frac{dx}{x-3} = \frac{1}{3(x-1)^3} + \log|x-3|$$

$$\int \frac{2x^4-10x^3+18x^2-15x+7}{(x-1)^4(x-3)}\,dx = \int\frac{dx}{x-1} - \int\frac{dx}{(x-1)^4} + \int\frac{dx}{x-3} = \frac{1}{3(x-1)^3} + \log|x^2-4x+3|$$

$$\int \frac{3x^4+6x^2-2x+1}{(x+1)(x^2+1)^2}\,dx = 3\int\frac{dx}{x+1} - 2\int\frac{dx}{(x^2+1)^2} = 3\log|x+1| - \operatorname{arctg} x - \frac{x}{x^2+1},$$

$$\int \frac{x^6+3x^4+3x^2-8x-15}{(x+2)(x^2+1)^3}\,dx = \int\frac{dx}{x+2} - 8\int\frac{dx}{(x^2+1)^3} = \log|x+2| - 3\operatorname{arctg} x - \frac{3x}{x^2+1} - \frac{2x}{(x^2+1)^2}$$

$$\int \frac{-x^4-3x^2+4}{(x^2+1)^4}\,dx = -\int\frac{dx}{(x^2+1)^2} - \int\frac{dx}{(x^2+1)^3} + 6\int\frac{dx}{(x^2+1)^4} = \operatorname{arctg} x + \frac{x}{x^2+1}\left(1+\frac{1}{x^2+1}+\frac{1}{(x^2+1)^2}\right)$$

$$\int \frac{x^4+x^3+x^2+3x+18}{(x+1)^2(x^2+2x+5)^2}\,dx = \int\frac{dx}{(x+1)^2} - \int\frac{3x+7}{(x^2+2x+5)^2}\,dx = -\frac{1}{x+1} - \frac{1}{4}\operatorname{arctg}\frac{x+1}{2} - \frac{x-2}{2(x^2+2x+5)}$$

11. Integrali di funzioni razionali con parte intera non nulla.

Negli esercizi del numero precedente la parte intera [1] delle funzioni razionali è il polinomio nullo, giacché il grado del numeratore è minore del grado del denominatore. Riportiamo ora alcuni integrali di funzioni razionali nelle quali il numeratore ha grado maggiore del denominatore [cfr. pag. 468].

$$\int \frac{3x^3+7x^2+4x-1}{3x^2+7x+2}\,dx = \frac{1}{2}x^2 + \log|x+2| - \frac{1}{3}\log|3x+1|$$

$$\int \frac{4x^3+4x^2-18x+11}{2x^2+5x-3}\,dx = x^2-3x+\log|x+3| + \frac{1}{2}\log|2x-1|$$

$$\int \frac{3x^4-10x^2+9x-4}{(x-1)^2}\,dx = x^3+3x^2-x+\log|x-1| + \frac{2}{x-1}$$

$$\int \frac{x^4+2x^3+9x^2+8x+2}{x(x^2+2x+2)}\,dx = \frac{1}{2}x^2 + \log|x| + 3\log(x^2+2x+2)$$

$$\int \frac{2x^4+x^3+7x^2+5x+2}{x(x^2+x+2)}\,dx = x^2-x+\log|x| + \frac{3}{2}\log(x^2+x+2) + \frac{9}{\sqrt{7}}\operatorname{arctg}\frac{2x+1}{\sqrt{7}}$$

$$\int \frac{3x^5+5x^3-2x^2+4x+2}{x^3-1}\,dx = x^3+5x+4\log|x-1| - \frac{3}{2}\log(x^2+x+1) - \sqrt{3}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}$$

$$\int \frac{12x^6+4x^5-9x^4-24x^3+24x^2+4x+5}{(2x-1)^2(x^2+2x+2)}\,dx = x^3-x^2-x-\frac{3}{2}\,\frac{1}{2x-1} + \operatorname{arctg}(x+1)$$

$$\int \frac{x^6-x^5+3x^4-2x^3+3x^2-3x-1}{(x+1)(x^2+1)^2}\,dx = \frac{1}{2}x^2-2x-\frac{x}{x^2+1} + 3\log|x+1| - \operatorname{arctg} x.$$

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[1] Cfr. pag. 380.

12. Negli integrali che seguono, anch'essi di funzioni razionali, conviene preventivamente effettuare una sostituzione del tipo $x^n = t$ [cfr. esempio alla fine di pag. 471].

$$\int \frac{x(x^2+1)}{(x^2-9)(x^2-4)}\,dx = \log|x^2-9| - \frac{1}{2}\log|x^2-4| \quad (1),$$

$$\int \frac{x\,dx}{(x^2-3)(x^2-4)} = \frac{1}{2}\log\left|\frac{x^2-4}{x^2-3}\right| , \quad \int \frac{x\,dx}{x^2(x^2+1)} = \frac{1}{2}\log\frac{x^2}{x^2+1} ,$$

$$\int \frac{3x^5+x^3}{(x^2+2)(x^4+1)}\,dx = \log(x^2+2) + \frac{1}{4}\log(x^4+1) - \frac{1}{2}\operatorname{arctg} x^2 \quad (2),$$

$$\int \frac{2x^7+x^5+2x^3-x}{x^8-1}\,dx = \frac{1}{2}\log|x^4-1| + \frac{1}{2}\operatorname{arctg} x^2$$

$$\int \frac{x^5+x^2}{(x^3-9)(x^3-4)}\,dx = \frac{2}{3}\log|x^3-9| - \frac{1}{3}\log|x^3-4| , \quad \int \frac{x^2\,dx}{x^6-7x^3+12} = \frac{1}{3}\log\left|\frac{x^3-4}{x^3-3}\right| ,$$

$$\int \frac{x^4\,dx}{x^{10}-7x^5+12} = \frac{1}{5}\log\left|\frac{x^5-4}{x^5-3}\right| , \quad \int \frac{3x^3+2x}{(x^4+1)^2}\,dx = \frac{1}{2}\operatorname{arctg} x^2 + \frac{1}{4}\,\frac{2x^2-3}{x^4+1} ,$$

$$\int \frac{3x^9+6x^5-2x^3+x}{(x^2+1)(x^4+1)^2}\,dx = \frac{3}{2}\log(x^2+1) - \frac{1}{2}\operatorname{arctg} x^2 - \frac{1}{2}\,\frac{x^2}{x^4+1} .$$

---

(1) Si osserva che l'integrando è un'espressione razionale di x^2 moltiplicata per x (a meno di un fattore costante, x è la derivata di x^2). Perciò conviene, prima della decomposizione in fratti semplici, effettuare la sostituzione $x^2 = t$ [che fornisce $2x\,dx = dt$, quindi $x\,dx = \frac{1}{2}dt$].

(2) Notiamo che, volendo decomporre in fratti semplici l'integrando senza effettuare alcuna sostituzione, bisogna scomporre x^4+1 nel prodotto di due trinomi di secondo grado.

Cogliamo l'occasione per ricordare che la scomposizione in fattori a coefficienti reali, di un polinomio a coefficienti reali, si può effettuare in termini pratici se si conoscono gli zeri del polinomio nel campo complesso [cfr. pag. 369]. Peraltro per il polinomio x^4+1 si può procedere come segue:

$$x^4+1 = (x^4+1+2x^2) - 2x^2 = (x^2+1)^2 - (x\sqrt{2})^2 = (x^2+1+x\sqrt{2})(x^2+1-x\sqrt{2}) .$$

13. Integrali del tipo

$$\int R(e^x)\,dx$$

con $R=R(y)$ funzione razionale [10.I pag. 476].

$$\int \frac{3e^{2x}+2e^x}{2e^{2x}+3e^x+1}\,dx = \log(e^x+1)+\frac{1}{2}\log(2e^x+1)$$

$$\int \frac{e^{2x}\,dx}{3e^{2x}-2e^x-1} = \frac{1}{4}\log|e^x-1|+\frac{1}{12}\log(3e^x+1),\quad \int \frac{e^{2x}+3e^x}{(e^x+1)^2}\,dx = \log(e^x+1)-\frac{2}{e^x+1},$$

$$\int \frac{e^{2x}+2e^x+1}{e^{2x}+e^x+1}\,dx = x+\frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2e^x+1}{\sqrt{3}},\quad \int \frac{e^x-1}{2e^x-1}\,dx = x-\frac{1}{2}\log|2e^x-1|,$$

$$\int \frac{e^{2x}+9e^x+9}{e^{2x}+3e^x+3}\,dx = 3x-\log(e^{2x}+3e^x+3)+\frac{6}{\sqrt{3}}\operatorname{arctg}\frac{2e^x+3}{\sqrt{3}},$$

$$\int \frac{e^{3x}+6e^{2x}+9e^x}{(e^x-3)(e^{2x}+3)}\,dx = 3\log|e^x-3|-\log(e^{2x}+3),$$

$$\int \frac{11e^{2x}-17e^x+12}{e^{2x}-3e^x+4}\,dx = 3x+4\log(e^{2x}-3e^x+4)+\frac{8}{\sqrt{7}}\operatorname{arctg}\frac{2e^x-3}{\sqrt{7}},$$

$$\int \frac{11e^{2x}-8e^x+3}{e^x(e^{2x}-2e^x+3)}\,dx = -2x+\log(e^{2x}-2e^x+3)-e^{-x}+\frac{8}{\sqrt{2}}\operatorname{arctg}\frac{e^x-1}{\sqrt{2}}$$

$$\int \frac{e^x\,dx}{(e^{2x}+1)^2} = \frac{1}{2}\left(\operatorname{arctg}e^x+\frac{e^x}{e^{2x}+1}\right),\quad \int \frac{4e^{3x}+9e^{2x}-18e^x}{(e^{2x}+4)(e^{2x}-3e^x+2)}\,dx = \log\frac{|e^x-1|(e^x-2)^2}{\sqrt{(e^{2x}+4)^3}}-\frac{1}{2}\operatorname{arctg}\frac{e^x}{2},$$

$$\int \frac{2e^{4x}+e^{3x}+7e^{2x}+5e^x+2}{e^{2x}+e^x+2}\,dx = e^{2x}-e^x+x+\frac{3}{2}\log(e^{2x}+e^x+2)+\frac{9}{\sqrt{7}}\operatorname{arctg}\frac{2e^x+1}{\sqrt{7}}.$$

14. Integrali del tipo:

$$\int R(\operatorname{sen}x,\cos x)\,dx \qquad [10.\text{II pag. } 476].$$

$$\int \frac{dx}{\operatorname{sen}x-\cos x-1} = \log\left|\operatorname{tg}\frac{x}{2}-1\right|,\quad \int \frac{dx}{\operatorname{sen}x-\cos x+1} = -\log\left|1+\operatorname{cotg}\frac{x}{2}\right| \quad (1)$$

---

(1) I due integrali si possono anche calcolare senza alcuna sostituzione, tenendo conto delle relazioni:

$$\operatorname{sen}x = 2\operatorname{sen}\frac{x}{2}\cos\frac{x}{2},\quad 1+\cos x = 2\cos^2\frac{x}{2},\quad 1-\cos x = 2\operatorname{sen}^2\frac{x}{2}$$

e dividendo numeratore e denominatore per $2\cos^2\frac{x}{2}$ nel primo integrale, per $2\operatorname{sen}^2\frac{x}{2}$ nel secondo.

$$\int \frac{dx}{1-\cos x-3\operatorname{sen} x} = \frac{1}{3}\log\left|1-3\cot g\frac{x}{2}\right| , \quad \int \frac{\operatorname{sen} x-\cos x-1}{\operatorname{sen} x\,(2\operatorname{sen} x-\cos x-1)}dx = \log\left|\operatorname{tg}\frac{x}{2}\right| - \frac{1}{2}\log\left|2\operatorname{tg}\frac{x}{2}-1\right| ,$$

$$\int \frac{dx}{3(1+\operatorname{sen} x)+\cos x} = \log\left|\frac{1+\operatorname{tg}\frac{x}{2}}{2+\operatorname{tg}\frac{x}{2}}\right| \; (^1), \quad \int \frac{1-\cos x+\operatorname{sen} x}{\operatorname{sen} x\,(8-4\cos x+7\operatorname{sen} x)}dx = \frac{1}{2}\log\left|2\operatorname{tg}\frac{x}{2}+1\right| - \frac{1}{3}\log\left|3\operatorname{tg}\frac{x}{2}+2\right| ,$$

$$\int \frac{4\operatorname{sen} x-3\cos x+3}{\operatorname{sen} x\,(7\operatorname{sen} x+\cos x+5)}dx = \log\left|\operatorname{tg}\frac{x}{2}+3\right| + \frac{1}{2}\log\left|2\operatorname{tg}\frac{x}{2}+1\right| ,$$

$$\int \frac{2\operatorname{sen} x+11\cos x+11}{(6\cos x+\operatorname{sen} x+6)^2}dx = 2\log\left|\operatorname{tg}\frac{x}{2}+6\right| + \frac{1}{\operatorname{tg}\frac{x}{2}+6} ,$$

$$\int \frac{\operatorname{sen} x+3\cos x+3}{(\operatorname{sen} x+\cos x+1)^2}dx = \log\left|\operatorname{tg}\frac{x}{2}+1\right| - \frac{2}{\operatorname{tg}\frac{x}{2}+1} ,$$

$$\int \frac{13\operatorname{sen} x-1-\cos x}{\operatorname{sen} x-4\cos x-4}dx = x+6\log\left|\operatorname{sen}\frac{x}{2}-4\cos\frac{x}{2}\right| .$$

15. Integrali del tipo:

$$\int R(\operatorname{tg} x)\,dx \qquad [\text{n.2 pag. 478}] .$$

$$\int \frac{1+\operatorname{tg}^2 x}{\operatorname{tg}^2 x-5\operatorname{tg} x+6}dx = \log\left|\frac{\operatorname{tg} x-3}{\operatorname{tg} x-2}\right| , \quad \int \frac{\operatorname{tg}^2 x+7}{\operatorname{tg} x-1}dx = 4\log|\operatorname{tg} x-1| + 3\log|\cos x| - 3x$$

$$\int \frac{13\operatorname{tg} x-1}{\operatorname{tg} x-4}dx = 3\log|\operatorname{sen} x-4\cos x| + x , \quad \int \frac{3\operatorname{tg}^2 x+\operatorname{tg} x}{\operatorname{tg} x+2}dx = 2\log|\operatorname{tg} x+2| - \log|\cos x| - x$$

$$\int \frac{3\operatorname{tg}^2 x+2\operatorname{tg} x-1}{\operatorname{tg} x+3}dx = 2\log|\operatorname{tg} x+3| - \log|\cos x| - x$$

$$\int \frac{2\operatorname{tg}^3 x+\operatorname{tg}^2 x-9\operatorname{tg} x-1}{(\operatorname{tg} x+2)^2}dx = 3\log|\operatorname{tg} x+2| - \frac{1}{\operatorname{tg} x+2} + \log|\cos x| - 2x$$

$$\int \frac{2\operatorname{tg}^3 x-13\operatorname{tg}^2 x-22\operatorname{tg} x-5}{(\operatorname{tg} x+1)^2}dx = 6\log|\operatorname{tg} x+1| - \frac{1}{\operatorname{tg} x+1} + 4\log|\cos x| - 12x$$

$$\int \frac{7\operatorname{tg}^2 x+28\operatorname{tg} x+11}{(\operatorname{tg} x+3)^2}dx = -2\log|\operatorname{sen} x+3\cos x| + \frac{1}{\operatorname{tg} x+3} + 2x$$

---

(1) Si noti che, detto X l'insieme di definizione della funzione integranda, i punti del tipo $\pi+2k\pi$ appartengono ad X ma in essi non ha senso la funzione a secondo membro: quest'ultima però si prolunga ivi per continuità, ponendola uguale a 0.

Pertanto in un intervallo $I\subset X$ cui appartenga uno di tali punti, ad es. il punto π, una primitiva della funzione integranda è la funzione:

$$F : x\in I \longrightarrow \begin{cases} \log\left|\dfrac{1+\operatorname{tg}\frac{x}{2}}{2+\operatorname{tg}\frac{x}{2}}\right| & \text{se } x\neq\pi \\ 0 & \text{se } x=\pi \end{cases}$$

Per la questione in generale si veda la parte iniziale di pag. 478.

$$\int \frac{(\operatorname{tg}x+1)(\operatorname{tg}^2x+1)}{(3\operatorname{tg}x+2)^2}\,dx = \frac{1}{9}\log|3\operatorname{tg}x+2| - \frac{1}{9}\,\frac{1}{3\operatorname{tg}x+2}$$

$$\int \frac{(\operatorname{tg}x+10)(\operatorname{tg}^2x+1)}{(\operatorname{tg}x+9)^2}\,dx = \log|\operatorname{tg}x+9| - \frac{1}{\operatorname{tg}x+9}$$

$$\int \frac{3\operatorname{tg}^4x+6\operatorname{tg}^2x-2\operatorname{tg}x+1}{(\operatorname{tg}x+1)(\operatorname{tg}^2x+1)}\,dx = 3\log|\operatorname{tg}x+1| - x - \operatorname{sen}x\cos x\,,$$

$$\int \frac{-\operatorname{tg}^4x-3\operatorname{tg}^2x+4}{(\operatorname{tg}^2x+1)^3}\,dx = \operatorname{sen}x\cos x\,(\cos^4x+\cos^2x+1)+x\,.$$

16. Integrali del tipo precedente, in cui figurano anche monomi di grado pari di senx e cosx, o solo monomi di questo tipo [n.2 pag. 478].

$$\int \frac{3\operatorname{tg}x+2}{3\operatorname{sen}x\cos x+\operatorname{sen}^2x+1}\,dx = \log|\operatorname{tg}x+1| + \frac{1}{2}\log|2\operatorname{tg}x+1|$$

$$\int \frac{dx}{2\operatorname{sen}^2x+\operatorname{sen}x\cos x} = \log\left|\frac{\operatorname{tg}x}{2\operatorname{tg}x+1}\right|\,,\quad \int \frac{dx}{2\operatorname{sen}^2x+\operatorname{sen}x\cos x-\cos^2x} = \frac{1}{3}\log\left|\frac{2\operatorname{tg}x-1}{\operatorname{tg}x+1}\right|\,,$$

$$\int \frac{\operatorname{tg}x}{4\operatorname{sen}^2x-2\operatorname{sen}x\cos x-1}\,dx = \frac{1}{4}\log|\operatorname{tg}x-1| + \frac{1}{12}\log|3\operatorname{tg}x+1|\,.$$

$$\int \frac{6\operatorname{tg}x-7}{(3\operatorname{sen}x-5\cos x)^2}\,dx = \frac{2}{3}\log|3\operatorname{tg}x-5| - \frac{1}{3\operatorname{tg}x-5}\,.$$

$$\int \frac{\operatorname{tg}x+3}{1+\operatorname{sen}2x}\,dx = \log|\operatorname{tg}x+1| - \frac{2}{\operatorname{tg}x+1}\,,\quad \int \frac{2\operatorname{tg}x-1}{3\operatorname{sen}^2x+1+4\operatorname{sen}x\cos x}\,dx = \frac{1}{2}\log|2\operatorname{tg}x+1| + \frac{1}{2\operatorname{tg}x+1}$$

$$\int \frac{5\operatorname{sen}2x-1}{\cos x\,(\operatorname{sen}x-2\cos x)}\,dx = 3\log|\operatorname{tg}x-2| + 4\log|\cos x| + 2x$$

$$\int \frac{3\operatorname{sen}^2x+\operatorname{sen}x\cos x+1}{\cos x\,(\operatorname{sen}x+2\cos x)}\,dx = 3\log|\operatorname{tg}x+2| - \log|\cos x| - x$$

$$\int \frac{5\cos^2x-4\operatorname{sen}x\cos x+1}{\cos x\,(\operatorname{sen}x-2\cos x)(1+\cos^2x)}\,dx = \log|\operatorname{tg}x-2| - \frac{4}{\sqrt{2}}\operatorname{arctg}\frac{\operatorname{tg}x}{\sqrt{2}}$$

$$\int \frac{3\operatorname{sen}^4x+6\operatorname{sen}^2x\cos^2x-2\operatorname{sen}x\cos^3x+\cos^4x}{\cos x\,(\operatorname{sen}x+\cos x)}\,dx = 3\log|\operatorname{tg}x+1| - x - \operatorname{sen}x\cos x$$

$$\int \frac{\operatorname{sen}^4x+3\operatorname{sen}^2x\cos^2x-4\cos^4x}{1+\operatorname{tg}^2x}\,dx = \operatorname{sen}x\cos x\,(\operatorname{sen}^2x-\cos^4x-2) - x\,.$$

17. Integrali del tipo:

$$\int R\left(x, \sqrt[n]{\frac{ax+b}{cx+d}}\right)dx \qquad [10.\mathrm{III}\ \text{pag. } 480].$$

$$\int \frac{\frac{1}{(x+1)^2}\sqrt{\frac{x+1}{2x+1}}}{1+\sqrt{\frac{x+1}{2x+1}}}\,dx = 2\sqrt{\frac{2x+1}{x+1}} - 2\log\left(1+\sqrt{\frac{2x+1}{x+1}}\right)$$

$$\int \frac{\sqrt{x}+3}{\sqrt{x}\,(2x-3\sqrt{x}-2)}\,dx = 2\log|\sqrt{x}-2| - \log(2\sqrt{x}+1)$$

$$\int \frac{\sqrt{x+1}+2}{\sqrt{x+1}\,(6x+5-\sqrt{x+1})}\,dx = \log\left|2\sqrt{x+1}-1\right| - \frac{2}{3}\log\left(3\sqrt{x+1}+1\right)$$

$$\int \frac{\sqrt[3]{x}-6}{6x\sqrt[3]{x}-5\sqrt[3]{x^2}-7x}\,dx = \frac{3}{2}\log|2\sqrt[3]{x}+1| - \log|3\sqrt[3]{x}-5|$$

$$\int \frac{\sqrt[3]{x+2}}{3(x+2)\sqrt[3]{x+2}-2x-4-\sqrt[3]{(x+2)^2}}\,dx = \frac{3}{4}\log|\sqrt[3]{x+2}-1| + \frac{1}{4}\log|3\sqrt[3]{x+2}+1|$$

$$\int \frac{7\sqrt{x}+1}{\sqrt{x}\,(7\sqrt{x}-1)^2}\,dx = \frac{2}{7}\log|7\sqrt{x}-1| - \frac{4}{7}\,\frac{1}{7\sqrt{x}-1}$$

$$\int \frac{\sqrt{x+3}+1}{(x+7)\sqrt{x+3}+4(x+3)}\,dx = 2\log(1+\sqrt{x+3}) + \frac{2}{1+\sqrt{x+3}}$$

$$\int \frac{\sqrt[3]{x}+3}{\sqrt[3]{x^2}\,(1+\sqrt[3]{x})^2}\,dx = 3\log|1+\sqrt[3]{x}| - \frac{6}{1+\sqrt[3]{x}}$$

$$\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} = 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log(1+\sqrt[6]{x})$$

$$\int \frac{dx}{\sqrt{x+2}-2\sqrt[3]{x+2}} = 2\sqrt{x+2} + 6\sqrt[3]{x+2} + 24\sqrt[6]{x+2} + 48\log\left|\sqrt[6]{x+2}-2\right|$$

$$\int \frac{\sqrt[6]{x}-3}{6x\sqrt[6]{x}-2\sqrt[6]{x^5}-x}\,dx = 3\log(2\sqrt[6]{x}+1) - 2\log|3\sqrt[6]{x}-2|$$

$$\int \frac{3+\sqrt{2x+5}}{\sqrt{2x+5}\,(4x+8-3\sqrt{2x+5})}\,dx = \log|\sqrt{2x+5}-2| - \frac{1}{2}\log(2\sqrt{2x+5}+1)$$

$$\int \frac{\sqrt[3]{\frac{3x+1}{5x+2}}+3}{\sqrt[3]{\left(\frac{3x+1}{5x+2}\right)^2}\left(1+\sqrt[3]{\frac{3x+1}{5x+2}}\right)^2}\cdot\frac{1}{(5x+2)^2}\,dx = 3\log\left|1+\sqrt[3]{\frac{3x+1}{5x+2}}\right| - \frac{6}{1+\sqrt[3]{\frac{3x+1}{5x+2}}}.$$

18. Integrali del tipo:

$$\int R(x, \sqrt{ax^2+bx+c})\, dx \qquad [10.\text{IV pag.} 481].$$

<u>Caso $a>0$.</u>

$$\int \frac{dx}{\sqrt{x^2+x+3}} = \log|2x+1+2\sqrt{x^2+x+3}|, \quad \int \frac{dx}{\sqrt{x^2+3x+5}} = \log|2x+3+2\sqrt{x^2+3x+5}|,$$

$$\int \frac{dx}{\sqrt{4x^2+x+1}} = \frac{1}{2}\log|8x+1+4\sqrt{4x^2+x+1}|, \quad \int \frac{dx}{\sqrt{9x^2+x+2}} = \frac{1}{3}\log|18x+1+6\sqrt{9x^2+x+2}|,$$

$$\int \sqrt{x^2+3}\, dx = \frac{3}{2}\log(x+\sqrt{x^2+3}) + \frac{1}{2}x\sqrt{x^2+3} \qquad (1)$$

$$\int \frac{dx}{2x+\sqrt{x^2+3}} = \frac{2}{3}\log|1+x^2+x\sqrt{x^2+3}| - \log(x+\sqrt{x^2+3})$$

$$\int \frac{dx}{3x+\sqrt{x^2+5}} = \frac{3}{8}\log|4x^2+5+4x\sqrt{x^2+5}| - \frac{1}{2}\log|x+\sqrt{x^2+5}|$$

$$\int \frac{dx}{3x+2\sqrt{x^2+1}} = \frac{3}{5}\log|5x^2+2+5x\sqrt{x^2+1}| - \log(x+\sqrt{x^2+1})$$

$$\int \frac{dx}{x-3\sqrt{x^2+5}} = -\frac{1}{8}\log|2x^2+15+2x\sqrt{x^2+5}| - \frac{1}{4}\log(x+\sqrt{x^2+5})$$

$$\int \frac{dx}{\sqrt{4x^2+2}-1} = \frac{1}{2}\log(2x+\sqrt{4x^2+2}) + \operatorname{arctg}(2x-1+\sqrt{4x^2+2})$$

$$\int \frac{dx}{\sqrt{9x^2+5}-2} = \frac{1}{3}\log(3x+\sqrt{9x^2+5}) + \frac{4}{3}\operatorname{arctg}(3x-2+\sqrt{9x^2+5})$$

$$\int \frac{dx}{\sqrt{4x^2+3}-2} = \frac{1}{2}\log(2x+\sqrt{4x^2+3}) + \log\left|\frac{2x-3+\sqrt{4x^2+3}}{2x-1+\sqrt{4x^2+3}}\right|$$

$$\int \frac{dx}{\sqrt{9x^2+8}-3} = \frac{1}{3}\log(3x+\sqrt{9x^2+8}) + \log\left|\frac{3x-4+\sqrt{9x^2+8}}{3x-2+\sqrt{9x^2+8}}\right|$$

$$\int \frac{dx}{\sqrt{4x^2+1}-1} = \frac{1}{2}\log(2x+\sqrt{4x^2+1}) - \frac{1}{2x-1+\sqrt{4x^2+1}}$$

$$\int \frac{dx}{\sqrt{9x^2+4}-2} = \frac{1}{3}\log(3x+\sqrt{9x^2+4}) - \frac{4}{3}\,\frac{1}{3x-2+\sqrt{9x^2+4}}$$

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(1) Per ottenere un integrale di funzione razionale più semplice di quello cui si perviene col procedimento generale, conviene procedere come segue: dopo aver integrato per parti col fattore differenziale dx, nell'integrale ottenuto far comparire al numeratore il radicando del denominatore, il che consente di riottenere l'integrale di partenza col segno meno (che poi si porterà a primo membro); l'integrale che rimane si calcola poi facilmente per sostituzione (col procedimento solito). L'opportunità dell'artificio è dovuta al fatto che il radicale non figura al denominatore, come negli integrali precedenti.