Lezione del 21-12 bis

$$\int \frac{5^{x+1}}{1+5^{2x}}\,dx = \int \frac{5\cdot 5^{x}}{1+5^{2x}}\,dx = 5\int \frac{5^{x}}{1+5^{2x}}\,dx$$

$$\frac{f'(x)}{1+f^2(x)} \qquad \frac{1}{\ln 5}\int \frac{5^{x}\ln 5}{1+5^{2x}}\,dx = \frac{5}{\ln 5}\,\operatorname{arctg} 5^{x} + c$$

$(5^x)^2$

$\operatorname{arctg} f(x)$

ALTRO METODO

$$\int \frac{5\cdot 5^{x}}{1+5^{2x}}\,dx \qquad \text{Pongo} \quad 5^x = t$$

$$x = \log_5 t$$

$$dx = \frac{1}{t\ln 5}\,dt$$

$$5\int \frac{\cancel{t}}{1+t^2}\cdot\frac{1}{\cancel{t}\ln 5}\,dt =$$

$$\frac{5}{\ln 5}\int \frac{1}{1+t^2}\,dt = \frac{5}{\ln 5}\,\operatorname{arctg} t + C$$

$$= \frac{5}{\ln 5}\,\operatorname{arctg} 5^x + C$$

$$\int \frac{3^{x+2}}{1+3^{2x}}\,dx$$

$$\int \frac{4\sin^3 x + \cos x + \sin 3x}{\sin x}\,dx$$

$$\int \left(\frac{4\sin^{\cancel{3}} x}{\cancel{\sin x}} + \frac{\cos x}{\sin x} + \frac{\sin 3x}{\sin x} \right) dx$$

$$\int \left(4\sin^2 x + \frac{\cos x}{\sin x} + \frac{\sin 3x}{\sin x} \right) dx$$

$$4\int \cancel{\sin x \cdot \sin x \, dx}$$

per parti

$$\ln|\sin x| + 3x + c$$

$$\int \frac{\cos x}{\sin x} dx = \ln|\sin x|$$

$$\int \frac{\sin 3x}{\sin x} dx = \int \frac{3\sin x - 4\sin^3 x}{\sin x} dx$$

$$\sin 3x = 3\sin x - 4\sin^3 x$$

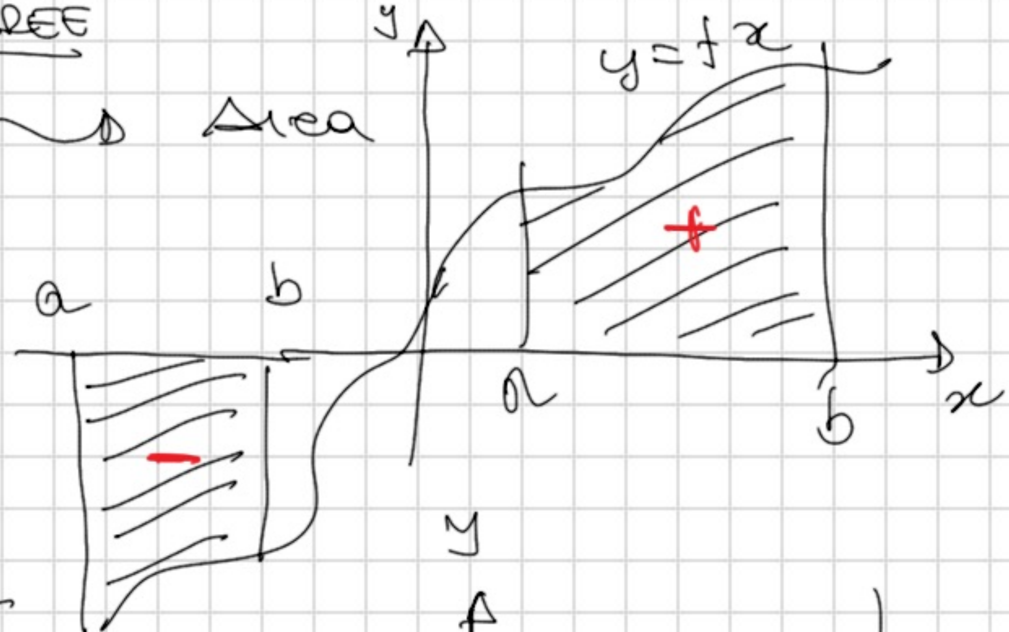
$\sin 3x = \sin(2x + x)$ Addizione ...

$$3\int \frac{\cancel{\sin x}}{\cancel{\sin x}} dx \quad - 4\int \frac{\sin^{\cancel{3}\,2} x}{\cancel{\sin x}} dx$$

$3x$

CALCOLO DI AREE

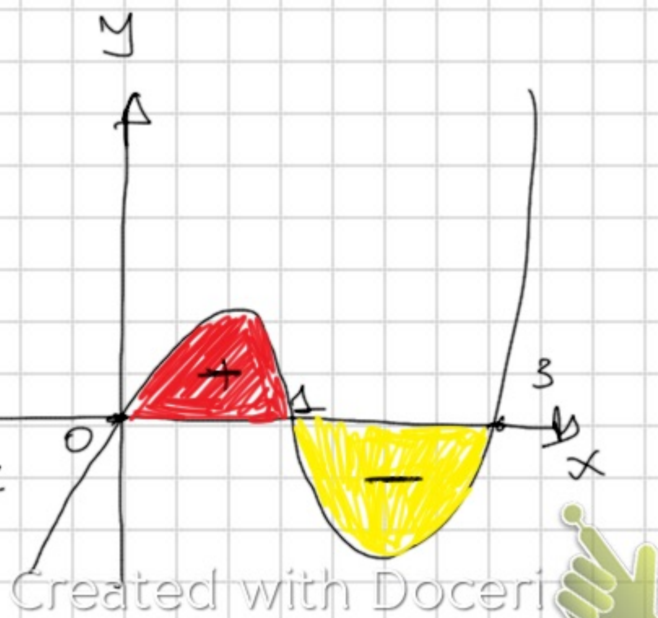
$$\int_a^b f(x)\,dx \rightsquigarrow \text{Area}$$



$$\cancel{\int_0^3 (x^3-4x^2+3x)\,dx}$$

NO

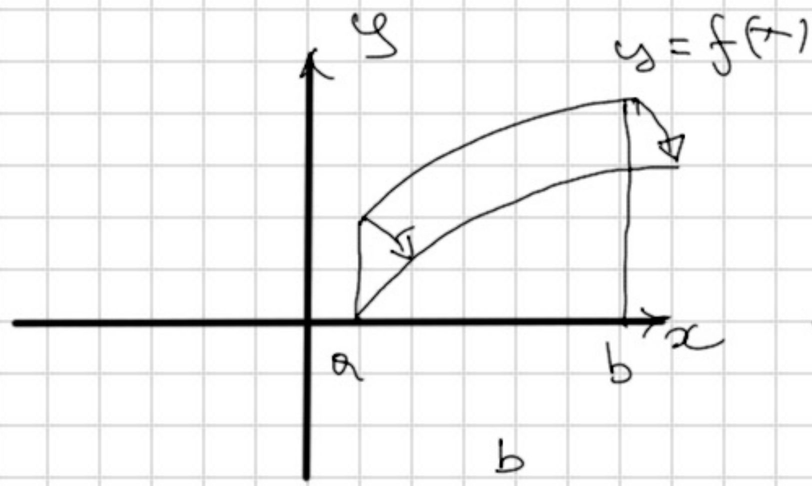
$$\int_0^1 (x^3-4x^2+3x)\,dx - \int_1^3 (x^3-4x^2+3x)\,dx$$

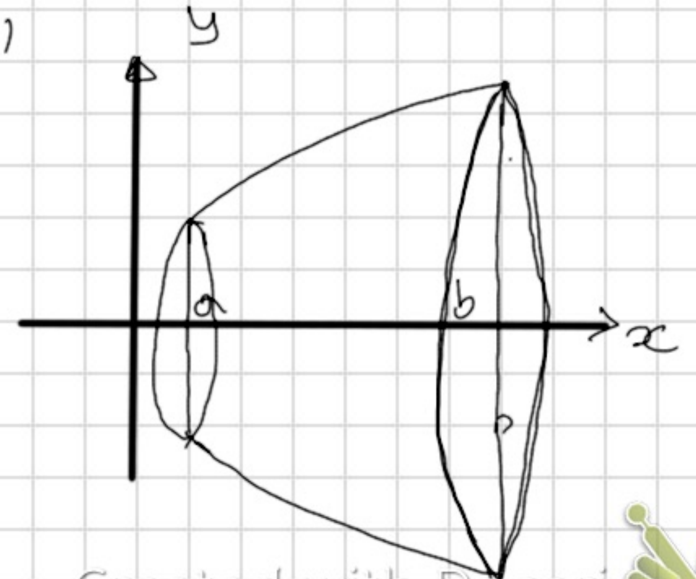


Solidi di rotazione

Data una funzione $y=f(x)$ continua in $[a,b]$ e non negativa e' il rettangoloide relativo ad $[a,b]$

Faccio ruotare il rettangoloide intorno all'asse x di un giro completo.
Ottengo un solido di rotazione

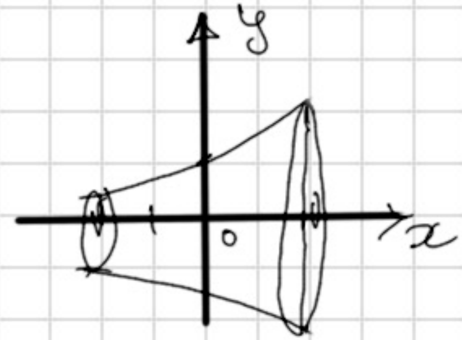




Volume $V = \pi \int_a^b f^2(x)\,dx$

$$y = e^{x}$$

Solido ottenuto
dalla rotazione
di $y = e^{x}$ intorno
all'asse x nell'intervallo
$[-1, 1]$



$$V = \pi \int_{-1}^{1} e^{2x}\,dx = \pi \frac{1}{2}\int_{-1}^{1} 2e^{2x}\,dx$$

$$= \left[\frac{\pi}{2} e^{2x}\right]_{-1}^{1} = \left(\frac{\pi}{2} e^{2} - \frac{\pi}{2} e^{-2}\right)$$

$$\int_{-1}^{1} \frac{2x^3 - 3x^2 - 10x + 9}{x+2}\, dx \qquad F(x)\ ?$$

$$\begin{array}{l|l} 2x^3 - 3x^2 - 10x + 9 & x+2 \\ \hline -(2x^3 + 4x^2) & 2x^2 - 7x + 4 \\ // \quad -7x^2 - 10x + 9 & \\ \quad -(-7x^2 - 14x) & \\ \quad // \quad 4x + 9 & \\ \quad\quad -(4x + 8) & \\ \quad\quad // \quad 1 & \end{array}$$

$$\int_{-1}^{1} \frac{1}{x+2}\, dx + \int_{-1}^{1} (2x^2 - 7x + 4)\, dx$$

$$\left[\ln|x+2|\right]_{-1}^{1} + \left[\frac{2}{3}x^3 - \frac{7x^2}{2} + 4x\right]_{-1}^{1}$$

$$\ln 3 - \cancel{\ln 1}^{\,0} + \left(\frac{2}{3} - \cancel{\frac{7}{2}} + \cancel{4} + \frac{2}{3} + \cancel{\frac{7}{2}} + \cancel{4}\right) = \ln 3 + \left(\frac{4}{3} + 8\right)$$

$$\int e^{2x} \operatorname{arctg}(e^{-x})\, dx$$

$$\operatorname{arctg}(e^{-x}) \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} \cdot \frac{1}{1+e^{-2x}}$$

$$-\frac{1}{2} \int \frac{e^{2x}}{1+e^{-2x}}\, dx$$

$$e^{x} = t$$

$$x = \ln t$$

$$dx = \frac{1}{t}\, dt$$

$$\int \frac{t^{\cancel{2}}}{1+\frac{1}{t^2}} \cdot \frac{1}{\cancel{t}}\, dt = \int \frac{t}{\frac{t^2+1}{t^2}}\, dt$$

$$= \int \frac{\cancel{t}}{t^2+1} \cdot \frac{1}{t^{\cancel{2}}}\, dt = \int \frac{1}{(t^2+1)t}\, dt$$

$$\int \frac{1}{(t^2+1)t}\,dt =$$

$$= \int \frac{A}{t}\,dt + \int \frac{Bt+C}{t^2+1}\,dt$$