



Kunnskap for en bedre verden

Design-based Research and/in/for Mathematics Education

Melih Turgut

Norwegian University of Science and Technology (NTNU)

Trondheim, Norway

06.10.2025



Design-based Research and/in/for Mathematics Education

Melih Turgut

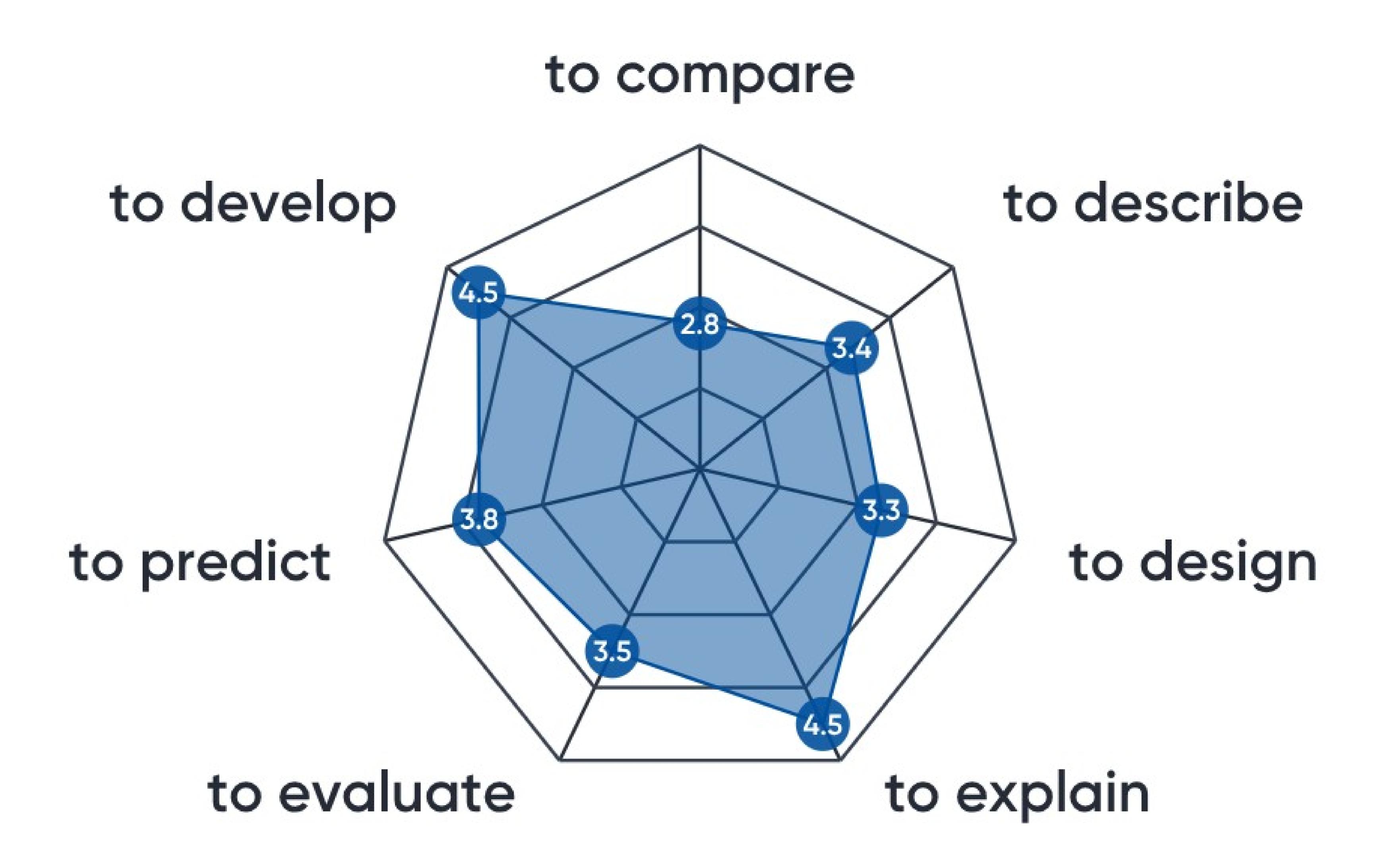
Norwegian University of Science and Technology, Trondheim, Norway

Email: melih.turgut@ntnu.no





Why we do "Research" in mathematics education?









Which (key)words prompt when I say "Design-based Research"?

```
mathematical engineering
                  new strategies
        cycle steam education
               project
             planning
            try and refine
```





Which theories, theoretical frameworks etc. you are familiar/working with?

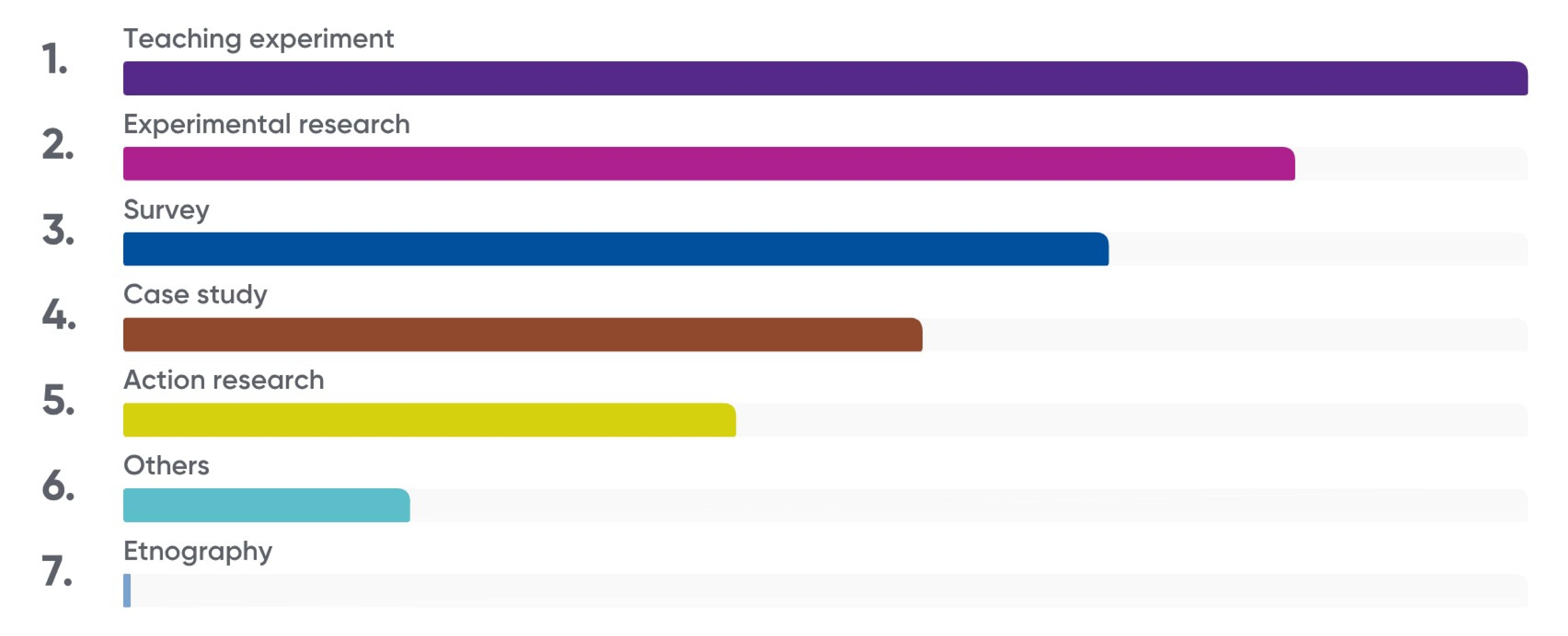
instrumental genesis role playing knowledge models problem solving







Which research method(s) you are familiar with?



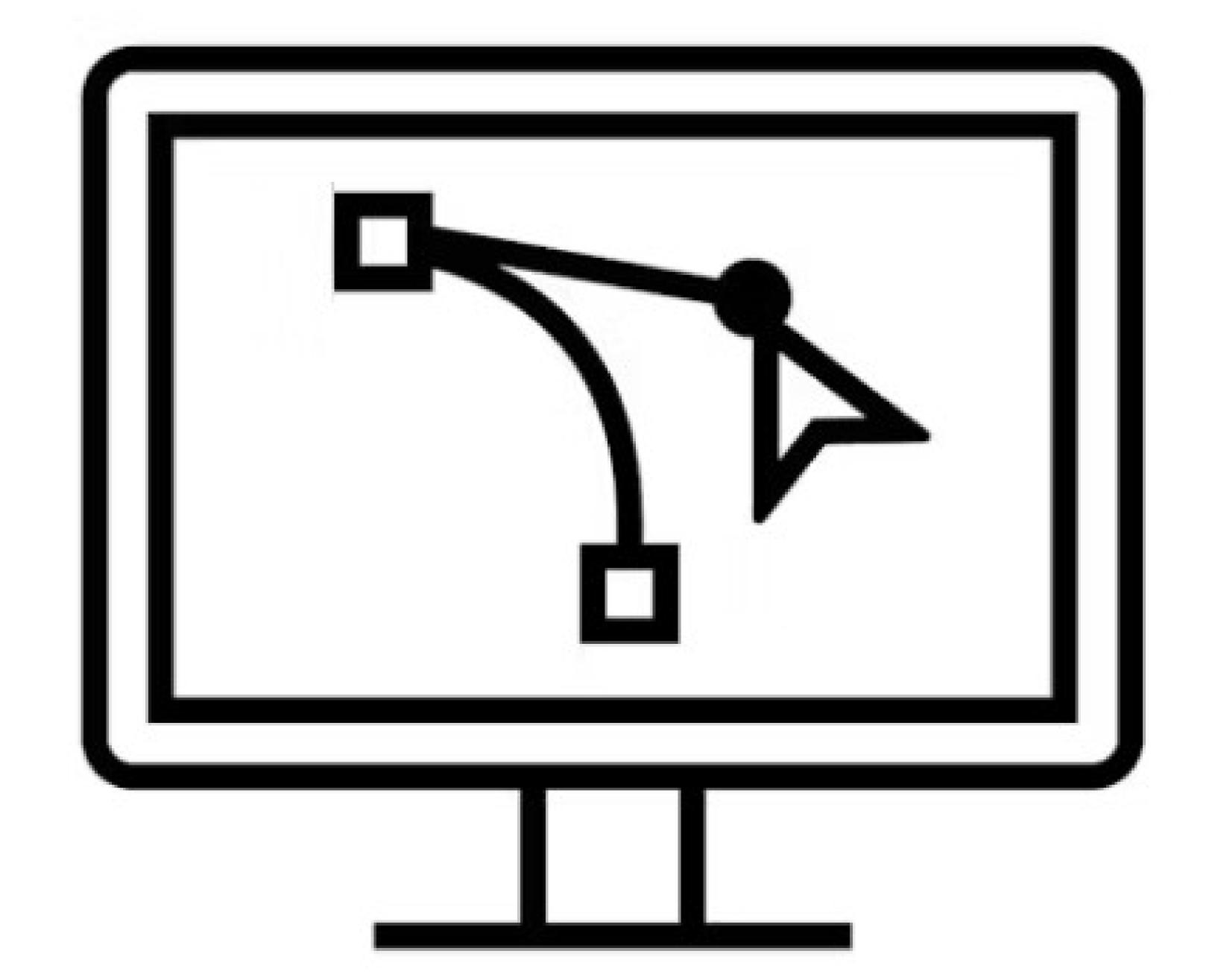






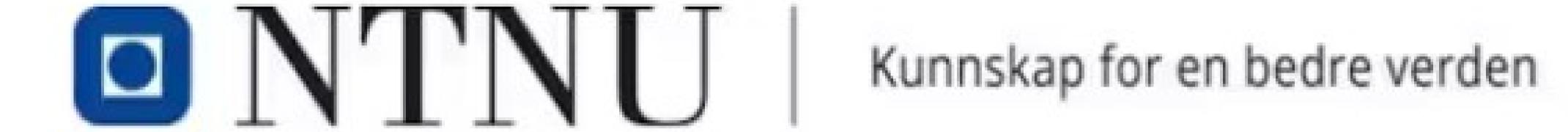
Outline

- Why we do research?
- Different but same
 - Design Research
 - Design-based Research (DBR)
 - Educational Design Research
 - Development Research
- Characteristics of DBR
 - Phases
 - Processes
 - Analyses
- An example
- Summary













Why we do «Research» in math education?

- Research (interrelated) aims/functions (Plomp, 2010, p.11)
 - to describe: e.g. what is the progress of classifying quadrilaterals . . .
 - to compare: e.g. what are the differences and similarities in ...
 - to evaluate: e.g. how well does a professional development program function in promoting inquiry-based teaching ...
 - to explain or to predict: e.g. what are the causes of poor performance in derivative ...
 - to design and develop: e.g. what are the characteristics of an effective teaching and learning strategy ...



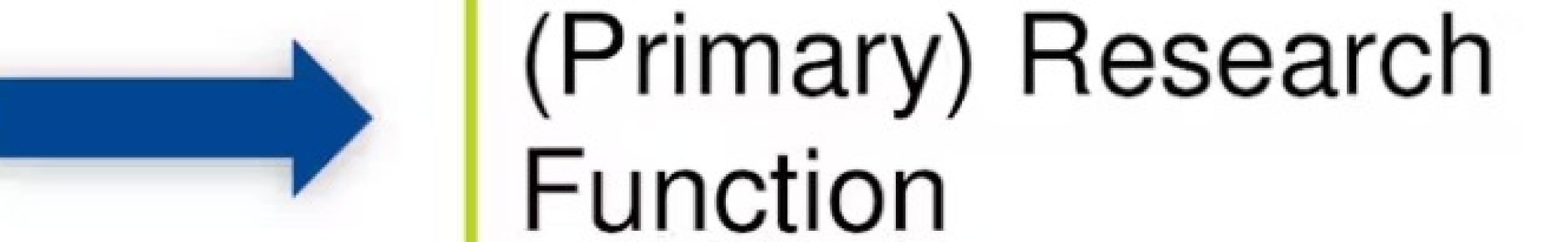




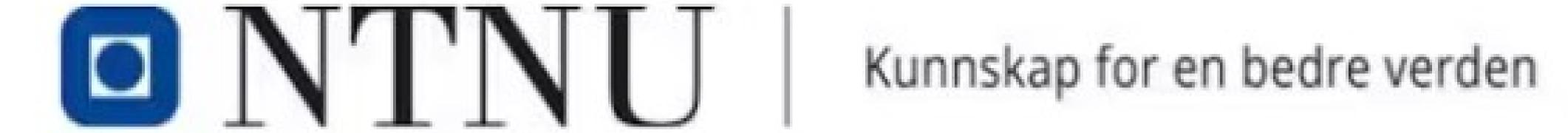
To point of departure

Chosen Framework

Research Focus, Research Question(s)



Research Methods



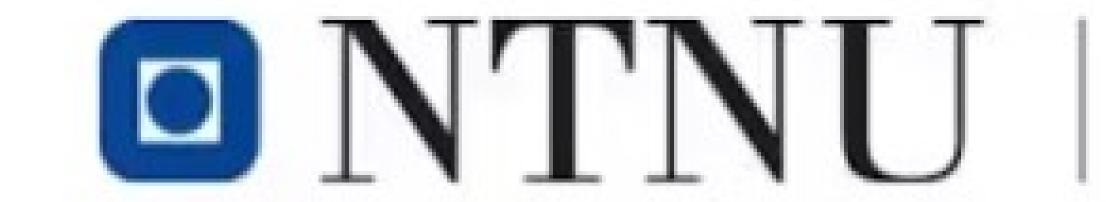




Let's have a particular focus on Design and Development

- Our focus is on
 - designing, testing and development of
 - a piece of curriculum segment (Msc and PhD),
 - a set of (prototype) tasks (Msc and PhD),
 - Local instruction theories (PhD, I will speak about at the end)
 - testing/using a theory/theoretical approach/model and refining the model
 - PhD, such as professional development programs

Design-based Research Methodology

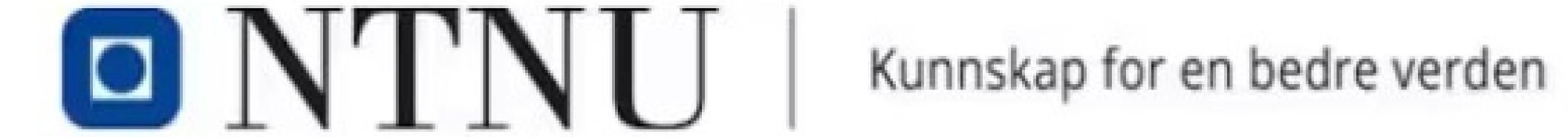






Note

- Design Research
- Design-based Research
- Developmental Research
- Educational Design Research
- - In this presentation, Design-based Research (DBR), which is wellestablished in mathematics education literature (see Bakker & van Eerde, 2015)



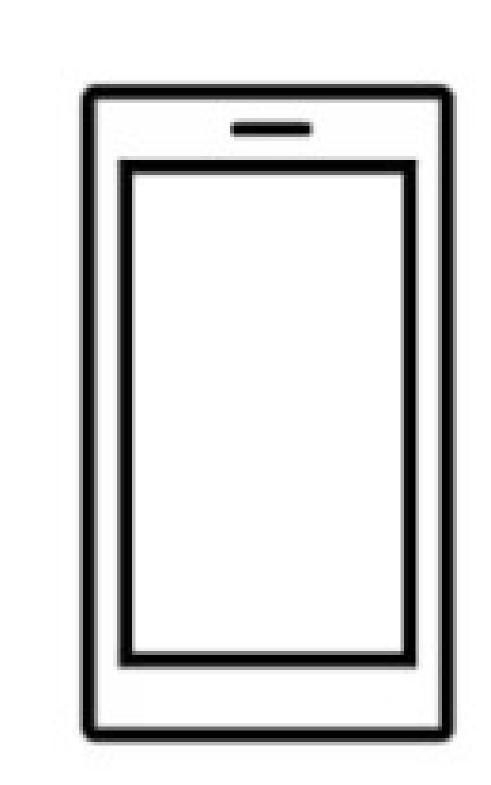


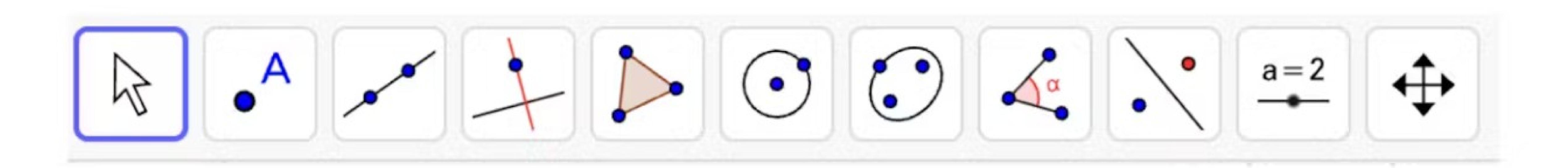


Key Characteristics of DBR (1)



- 1. Integration of <u>Design</u> and <u>Research</u>
 - Prototyping, testing and development of
 - A tool, for example, Geometer's Sketchpad, GeoGebra





Why this toolbar seems like this?







Key Characteristics of DBR (2)

- 2. The Role of <u>Hypotheses</u> and the <u>Engineering Nature</u> of DBR
 - Hypothesize-based,
 - such as, under a literature synthesis and theoretical framework, our design will likely promote student learning

The need for DBR: Why we design prototype tasks/tools/technologies in mathematics education?







The need for DBR

The need for DBR: Why we design prototype tasks/tools/technologies in mathematics education?



Innonative and Engineering

Overviewing existing tasks/tools/technologies What misses and why? Which hypotheses I can establish?

Why it should work?

Risks?



A careful literature review and framework guide these Kunnskap for en bedre verden questions and this process.



Key Characteristics of DBR (3)

3. Open and Interventionist Nature of DBR

	Naturalistic	Interventionist				
Closed	Survey: questionnaires with closed questions	Experimental Research (randomized controlled trial)				
Open	Survey: interviews with open questions Ethnography	 Action research Design-based research 				

Resource: Bakker and van Eerde (2015, p. 433)

Naturalistic vs. Interventionist and Open vs. Closed Research Approaches

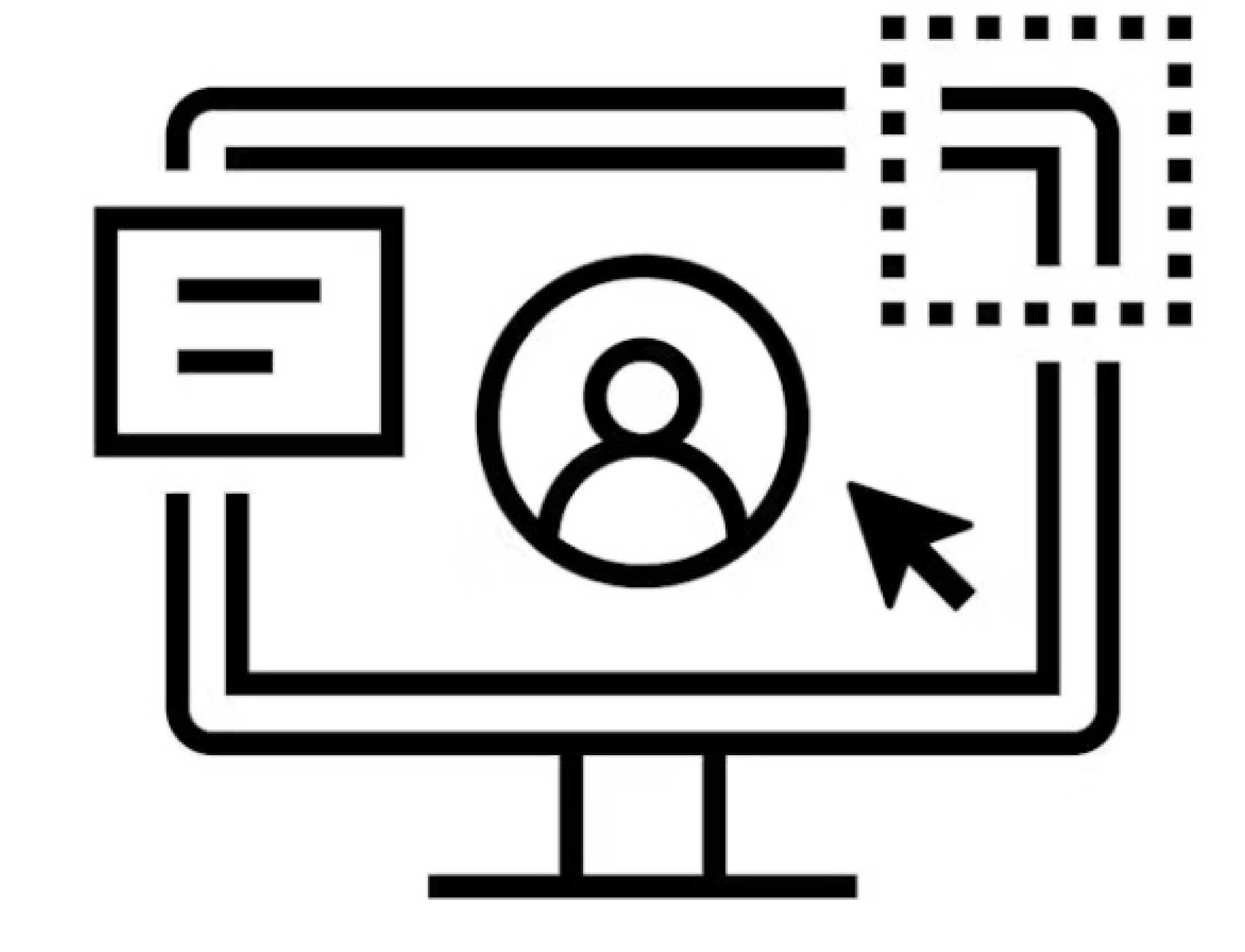


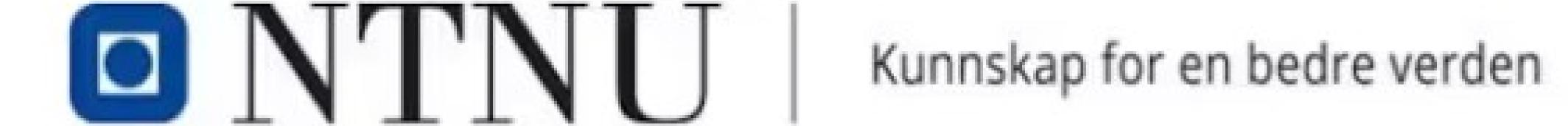




Open and Interventionist Nature of DBR

- Task/tool/technology prototype design
 - Classroom implementation
 - Trying to understand
 - What works and why
 - What does not work and why
 - Which parts deviated
 - Issues, hiccups

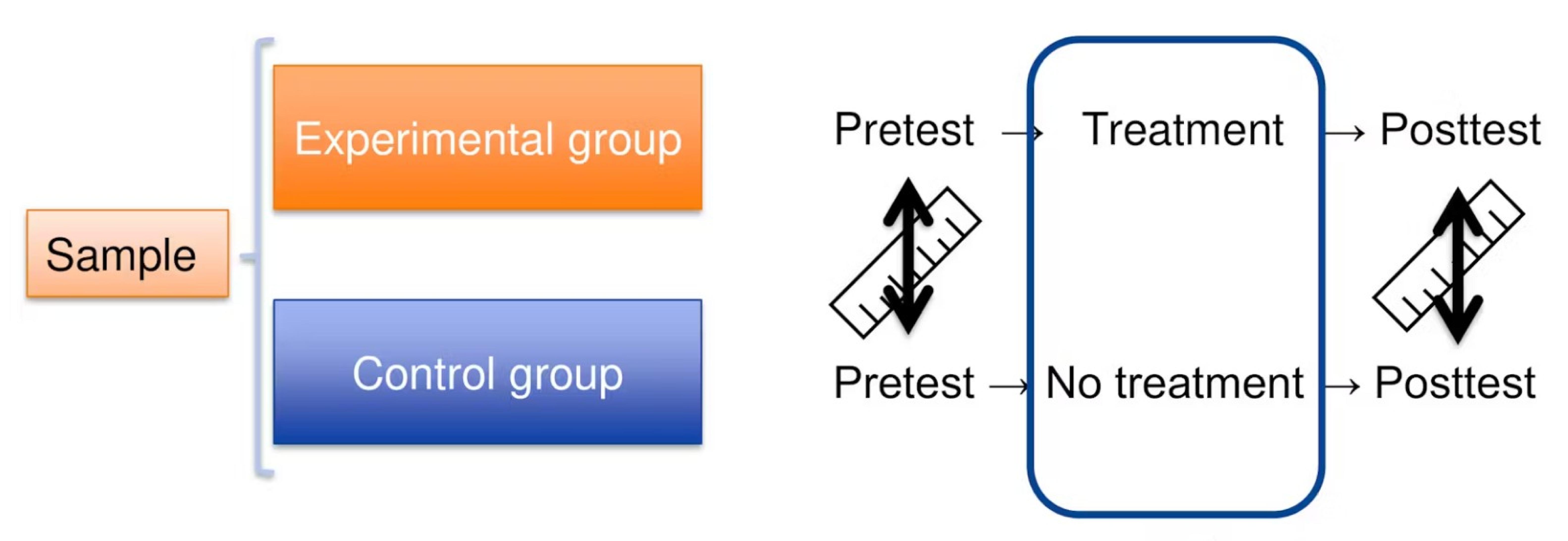








Reminder: Experimental research



What learners think mathematically in this process?

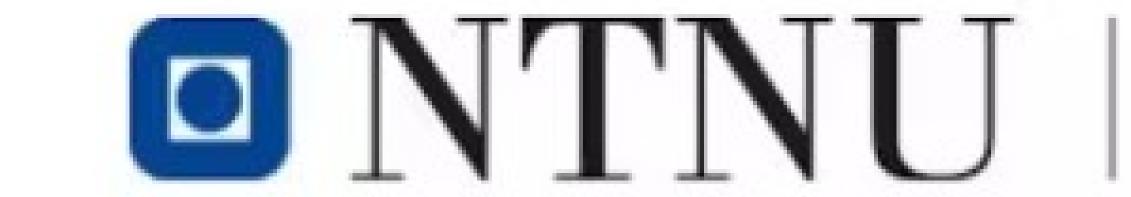






Experimental Research vs. DBR

Experimental Research	DBR			
Testing theory	Developing and testing theory simultaneously			
Comparison of existing teaching methods by means of experimental and control groups	Design of an innovative learning environment			
Proof of what works	Insight into how and why something works			
Research interest is isolated by manipulating variables separately	Holistic approach			
Statistical generalization	Analytic or theoretical generalization, transferability to other situations			

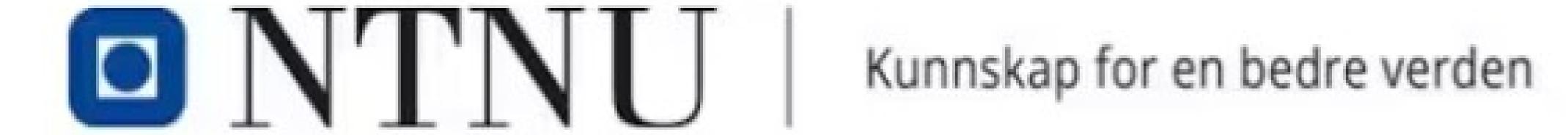


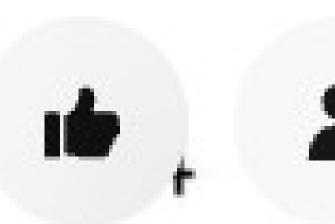


Action Research vs. DBR

	DBR Action Research						
Commonalities	Open, interventionist, researcher can be participant, reflective cyclic process						
Differences	Researcher can be observerDesign is necessary	Researcher can only be participantDesign is possible					

Resource: Bakker and van Eerde (2015, p. 435)







Summary of Key Characteristics

- Interventionist
- Testing and developing
 - theories/ approaches/ tools/ technologies/tasks
- Cyclic nature
 - Prototyping, testing, refining, re-testing, refining, developing, finalizing...





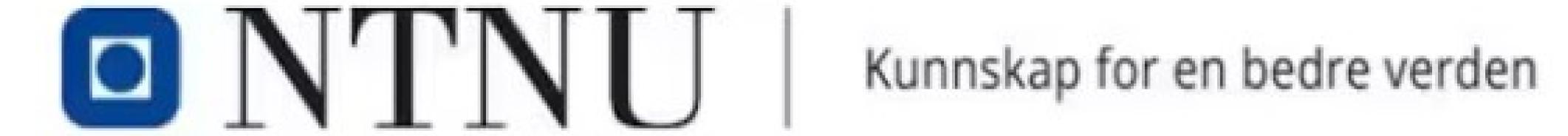


Phases of DBR

- Ok, I will consider it, where I can start?
- Three phases
 - Preparation and design
 - Teaching experiments
 - Retrospective analyses





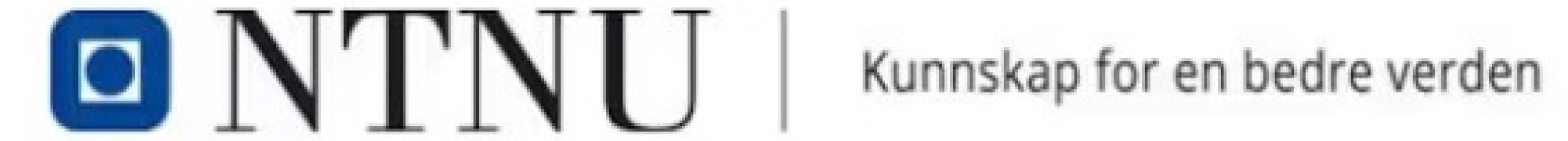






Preparation and Design (1)

- Some exemplary questions for start:
 - What is the (learning) problem? What causes the problem? Epistemological and ontological issues? (Literature Review)
 - Which topic? And why? (Literature Review)
 - Historical analysis? How mathematicians came up with this topic and why? What did they reinvent? (Literature Review and communicating with experts)
 - How to design prototype(s)? (the Need for a Framework)
 - How do textbooks introduce this topic? Does it work? Or not, why?







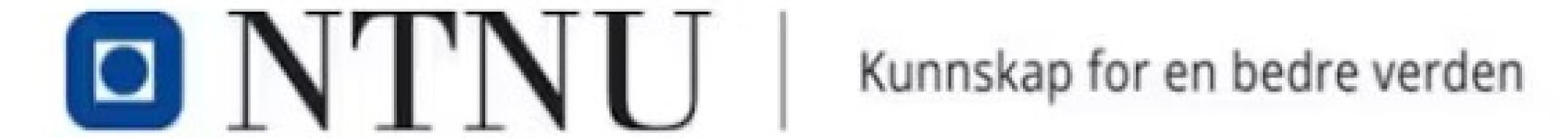
Preparation and Design (2)

- Gravemeijer (1994) characterizes the design researcher as a tinkerer or, in French, a bricoleur, who uses all the material that is at hand,
 - including theoretical insights (e.g., the need for a framework) and
 - practical experience with teaching and designing (e.g., literature review, textbook analysis, historical analysis).













Summary of «Preparation and Design»

- First, we need to justify why we need to design something **new**, by overviewing existing
 - Tools/ technologies,
 - Textbooks,
 - Models,
 - Frameworks...
- Designing (Prototyping) and Expert Review
 - How? (We have other tools)





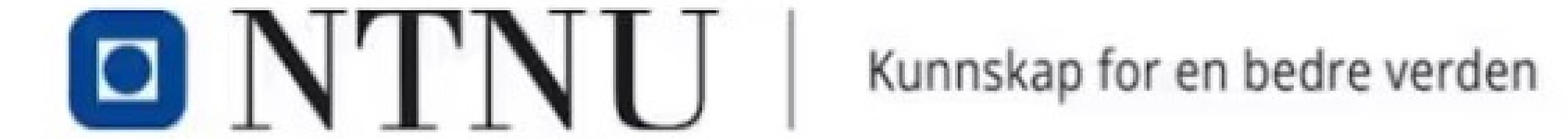
The most

critical part



Phase 2: Teaching Experiments

- After prototyping and expert review
 - Testing the designed taks/curriculum segment/ tools etc. in
 - Small groups (at the beginning)
 - Later in the classroom
 - Data collection and triangulation are crucial:
 - Task-based interviews, individually, in pairs, groups etc. (according to adopted theory and methods)
 - Observation notes
 - Surveys, open-ended questions
 - . . .







Phase 3: Retrospective Analyses

Mainly

- Coding and/or using an analysis tool (developed through the adopted theory and literature review) to code
 - Interviews, observations, the classroom dynamics/interaction and all

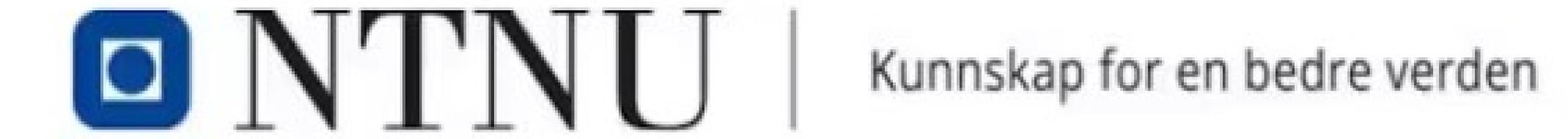
• Search for

- Which parts of the design
 - Partially worked and why,
 - Did not work and why,
 - Worked and why.

+				X	X			X	X	X	X	X		X	X	X	X	X	X
±	x		x										x						
_		X				X	X												
Task:	5d	5f	6a	6c	7	8	9c	9e	10b	11c	15	17	23b	23c	24a	24c	25d	34a	42

Note: an x means how well the conjecture accompanying that task matched the observed learning (- refers to confirmation for up to 1/3 of the students, and + to at least 2/3 of the students)

Resource: Bakker and van Eerde (2015, p. 442)







An Example

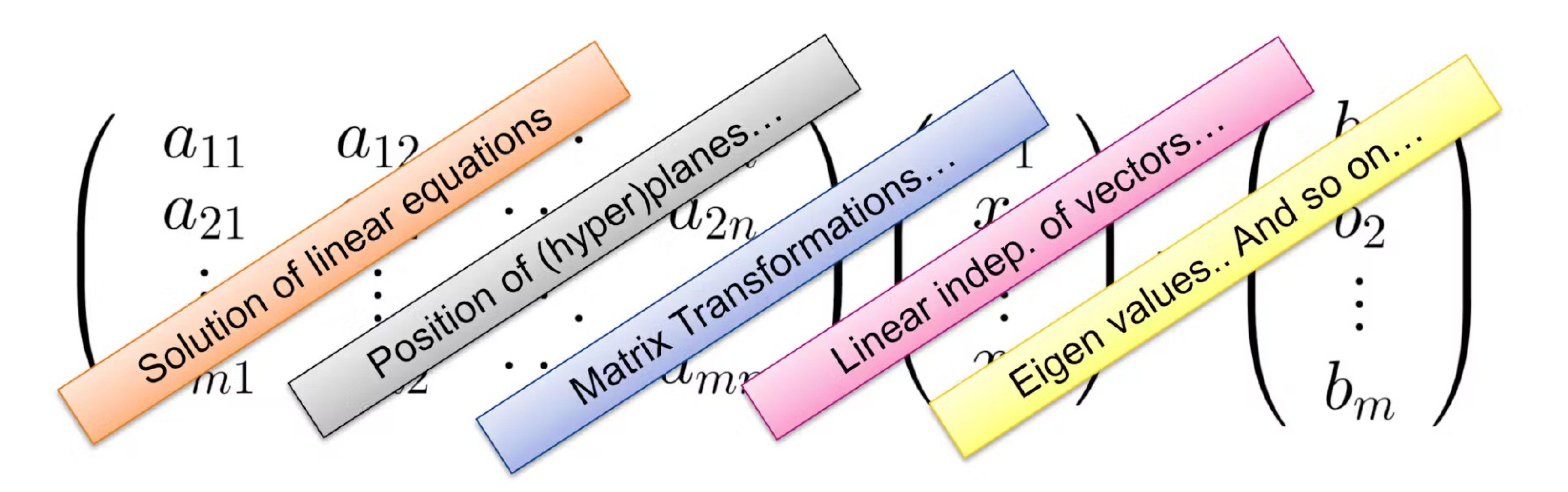
Click on the following link to access the task:

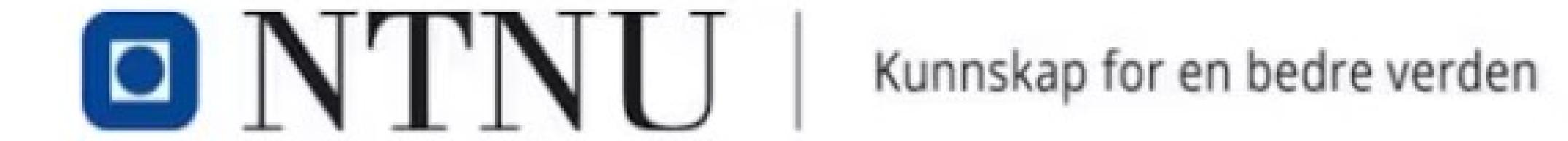
https://www.geogebra.org/m/bcynjm5s

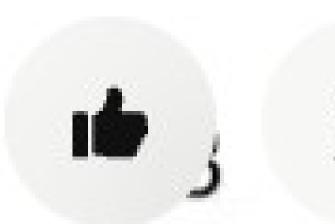




How this task was designed? Why?







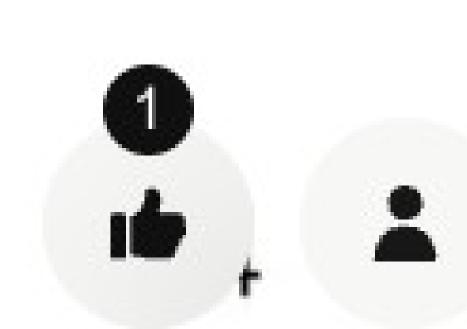


Phase 1: Design and Preparation

- Students are challenged with formal mathematical structure of the notion of transformation
- Initial Question: How can we design an environment to make a smooth transition?

 After this, my question changed ©
 - Literature Review -----
 - Linking functions and (linear) transformations are not easy.
 - Students often think linear transformations (LT) are just two algebraic rules.
 - Researchers suggest use of geometry at the beginning.







Textbook analysis

LT Linear Transformations

Definition LT. Linear Transformation. A *linear transformation*, $T: U \to V$, is a function that carries elements of the vector space U (called the *domain*) to the vector space V (called the *codomain*), and which has two additional properties

1.
$$T(\mathbf{u}_1 + \mathbf{u}_2) = T(\mathbf{u}_1) + T(\mathbf{u}_2)$$
 for all $\mathbf{u}_1, \mathbf{u}_2 \in U$.

2.
$$T(\alpha \mathbf{u}) = \alpha T(\mathbf{u})$$
 for all $\mathbf{u} \in U$ and all $\alpha \in \mathbb{C}$.

http://linear.ups.edu/fcla/section-LT.html

II.1 Definition

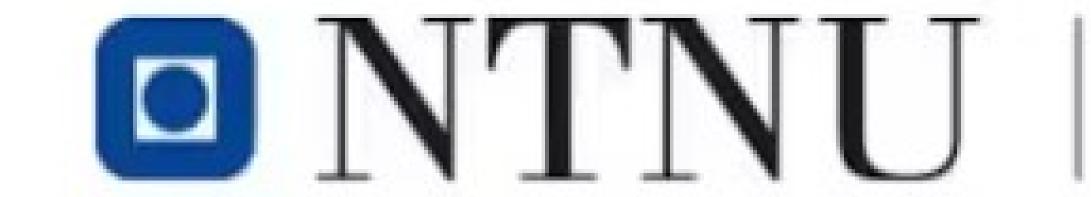
1.1 Definition A function between vector spaces h: $V \rightarrow W$ that preserves addition

if
$$\vec{v}_1, \vec{v}_2 \in V$$
 then $h(\vec{v}_1 + \vec{v}_2) = h(\vec{v}_1) + h(\vec{v}_2)$

and scalar multiplication

if
$$\vec{v} \in V$$
 and $r \in \mathbb{R}$ then $h(r \cdot \vec{v}) = r \cdot h(\vec{v})$

is a homomorphism or linear map.







Historical Development of LTs

Kleiner (2007, p. 82)

History of Linear Algebra

Functions – two variables

Geometric Transformations

> Expressing inputs and outputs

> > ... Formalization for vector spaces

5 History of Linear Algebra

5.3 Matrices and linear transformations

Matrices are "natural" mathematical objects: they appear in connection with linear equations, linear transformations, and also in conjunction with bilinear and quadratic forms, which were important in geometry, analysis, number theory, and physics.

Matrices as rectangular arrays of numbers appeared around 200 BC in Chinese mathematics, but there they were merely abbreviations for systems of linear equations. Matrices become important only when they are operated on—added, subtracted, and especially multiplied; more important, when it is shown what use they are to be put to.

Matrices were introduced implicitly as abbreviations of linear transformations by Gauss in his Disquisitiones mentioned earlier, but now in a significant way. Gauss undertook a deep study of the arithmetic theory of binary quadratic forms, f(x, y) = $ax^2 + bxy + cy^2$. He called two forms f(x, y) and $F(X, Y) = AX^2 + BXY + CY^2$ "equivalent" if they yield the same set of integers, as x, y, X, and Y range over all the integers (a, b, c) and A, B, C are integers). He showed that this is the same as saying that there exists a linear transformation T of the coordinates (x, y) to (X, Y) with determinant = 1 that transforms f(x, y) into F(X, Y). The linear transformations were represented as rectangular arrays of numbers—matrices, although Gauss did not use matrix terminology. He also defined implicitly the product of matrices (for the 2×2 and 3×3 cases only); he had in mind the composition of the corresponding linear transformations. See [1], [7], [16].

Linear transformations of coordinates, $y_j = \sum_{k=1}^n a_{jk} x_k (1 \le j \le m)$, appear prominently in the analytic geometry of the seventeenth and eighteenth centuries (mainly for $m = n \le 3$). This led naturally to computations done on rectangular varrays of numbers (a_{ik}) . Linear transformations also show up in projective geometry, (Grassman, and Peano's followers) founded in the seventeenth century and described analytically in the early nineteenth. See [2], [9].





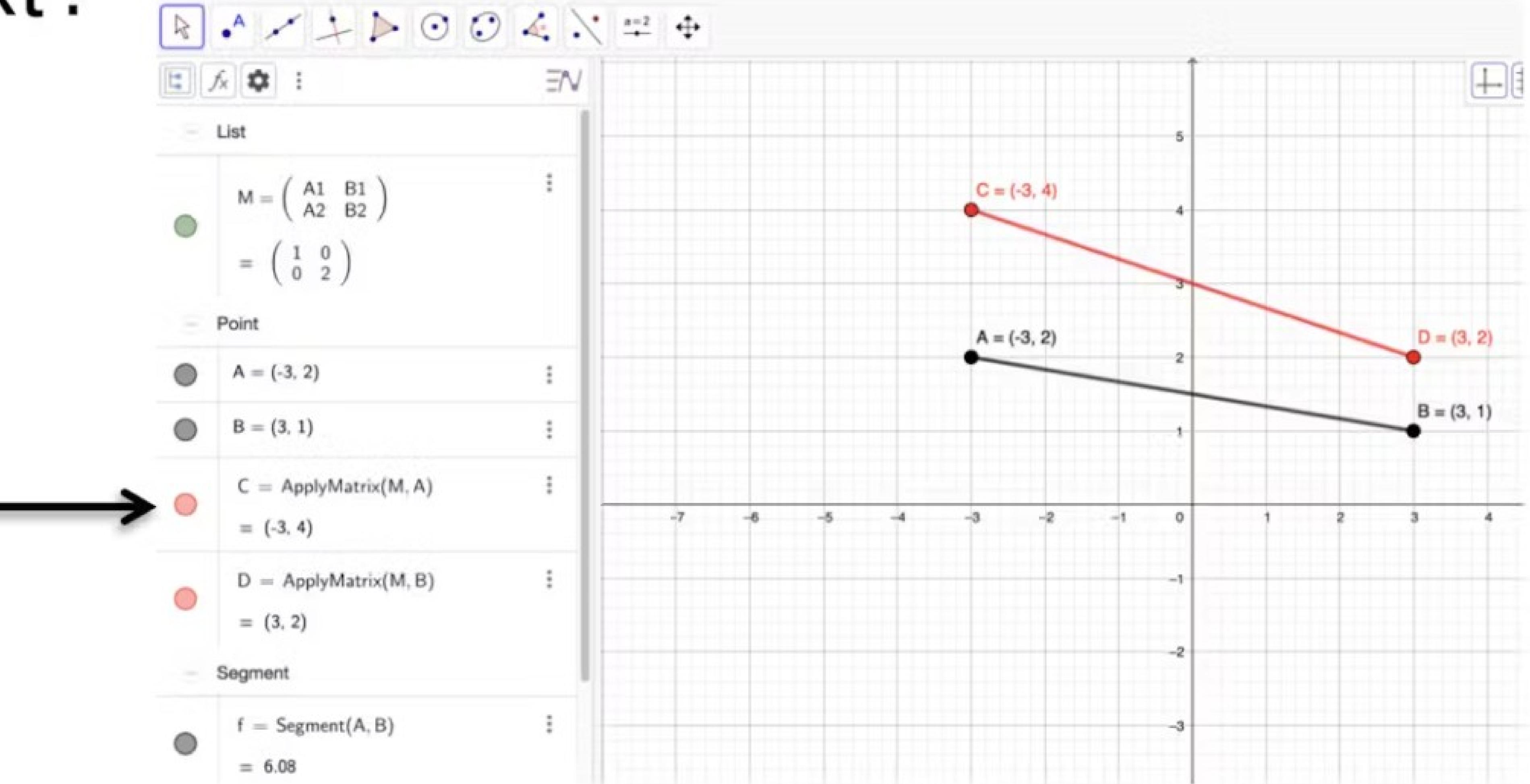


Phase 1: continued

 How to design a task where learners move from a function context to inputs, outputs and then a geometric

transformation context?

Literature and
Theory of Semiotic
Mediation guided
the whole process

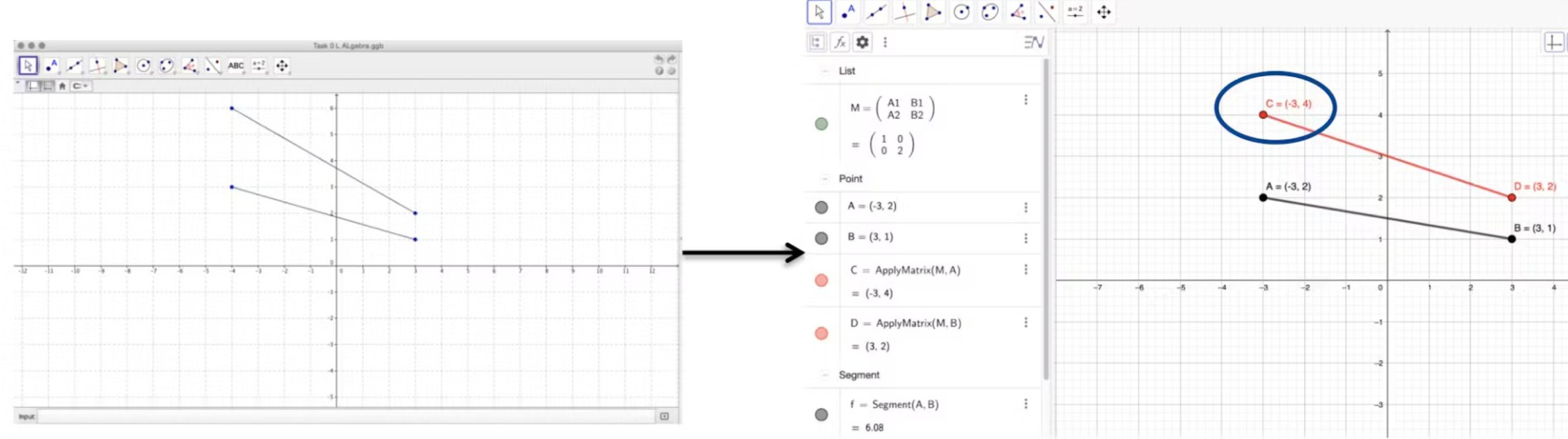








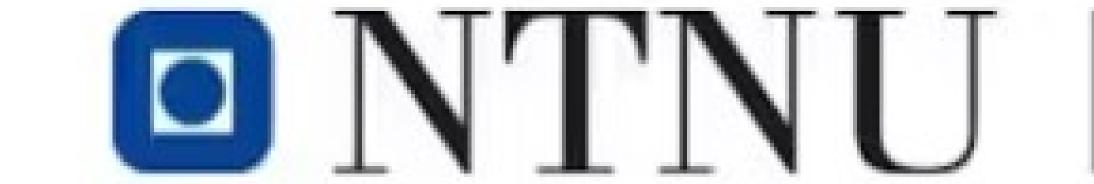
Phase 2: Teaching Experiments



Pilotting (1) with student pairs

Result: without coordinates, it did not work

Pilotting 2: Colorful line segments worked better. But they challenged with formulating functions







Finalization of the task

• This time

- Before task implementation
 - I reminded Calculus and multivariable functions, how we represent them.
 - I also revisited systems of linear equations, how we represent in vector-matrix form, to represent linear expressions

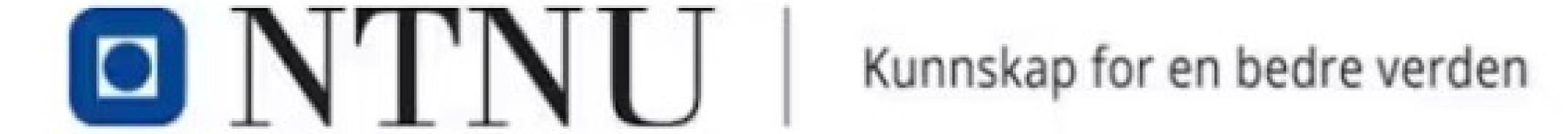
$$\frac{A}{1}$$
 $\frac{C}{1}$ \frac{C}

Fig. 12 Students' list of coordinates regarding the task

$$f:(x,y) \to (x+y,2y)$$
 $f(x+y) \to (x+y,2y)$
(a)

Fig. 13 a Tina's and b Sara's results regarding the transformation

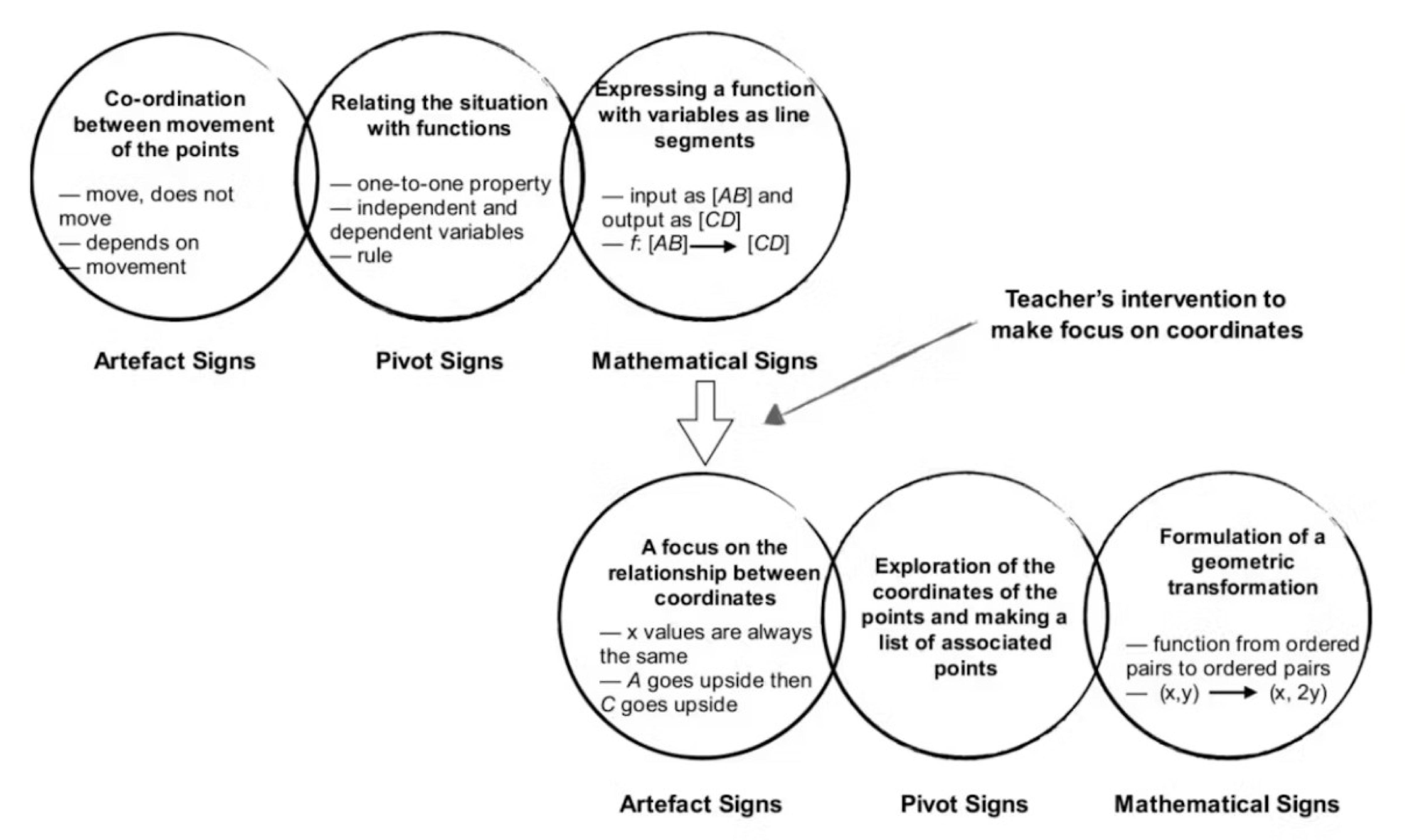
$$[1,1] \cdot \begin{bmatrix} x \\ y \end{bmatrix}_{2x_1} = \begin{bmatrix} x+y \\ 2y \end{bmatrix}_{2x_1} = 8$$







Phase 3: Retrospective Analyses





(a) 1827 Accesses (b) 20 Citations Explore all metrics →

Turgut (2019)





Avoid common mistakes on your

manuscript.



Note

- Indeed, it was a task sequence
 - From non-linear transformations to singular matrix transformations and to matrix transformation of a sphere.
 - Designed, tested, refined, tested, refined, and finalized.

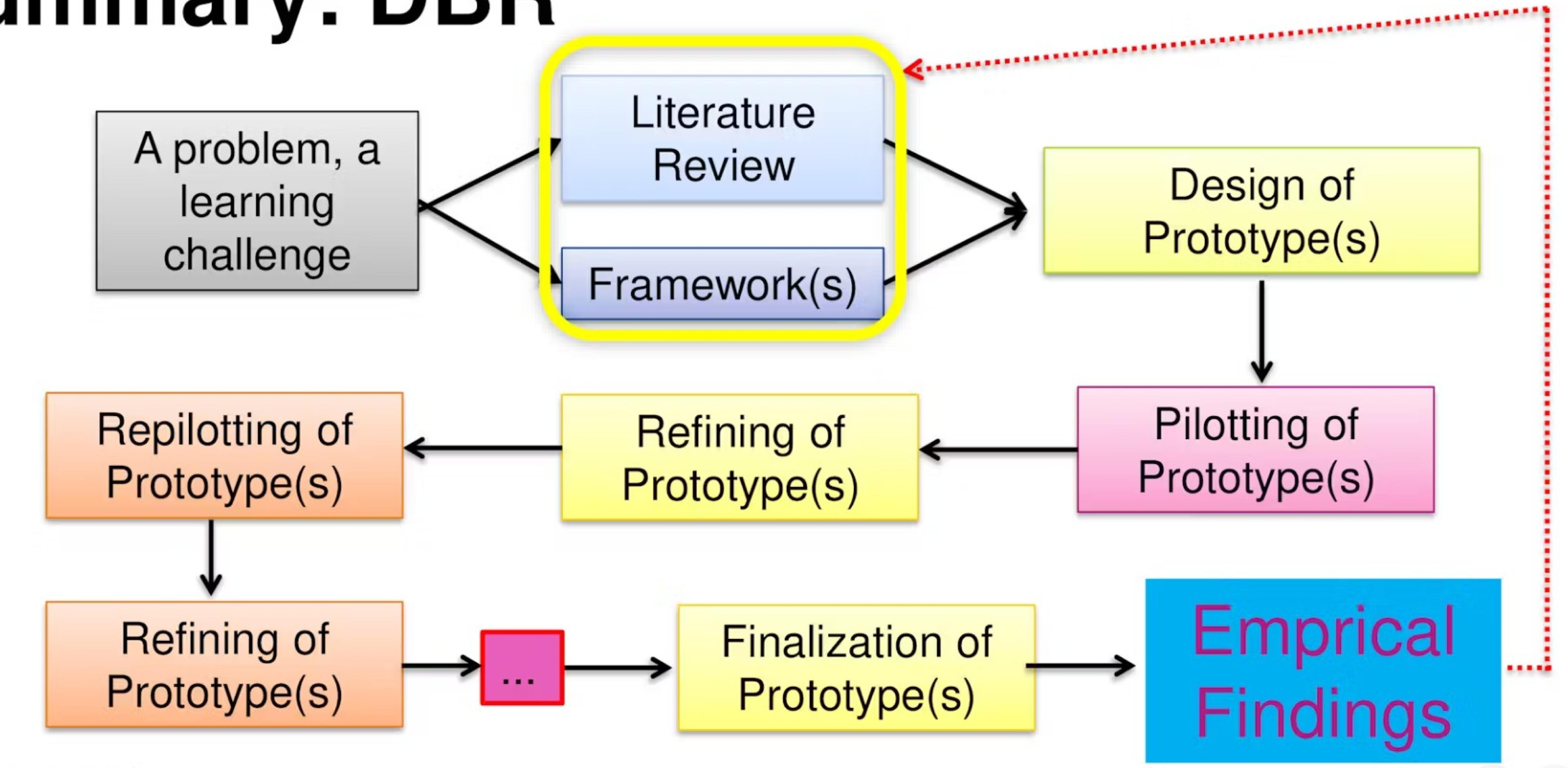
Associated Concepts	Goals with associated semiotic features (Technical part)	Exemplary Task	Four levels of Activity	Epistemic Artefacts as in DGEs (Semiotic potentials)	(Expected) Discursive and operational mathematical meanings (Cognitive part)	
- Variable - Independent and Dependent Variables - Function - Transformation	- Experience covariation between two quantities - Exploring dependency - independency relationships in algebraic and geometric contexts - Making connection between geometric figures and fundamentals of functions	- Explore and try to analyse the situations on the interface, validate your assertions (Task 1)	- Situational - Referential	- Dragging - Grid function - ApplyMatrix construction tool - Graphics window?	- Reasoning on the moveable points and line segments -Finding "dependency" - Comprehending "(In)Dependent variables" as line segments - Comprehending inputs as elements of a (geometric) domain and so "ordered pairs" - Function with "new" kind variables "transform(s) to"	
- Exploring different transformations in algebraic and geometric contexts - Linear - Analysing different situations' mathematics and forming new functions - Analysing output equations - Characterizing		- Explore and try to analyse the situations on the interface - Form a new mathematical situation (Task 2)	- Situational - Referential - General - Formal	- Dragging - Grid function - ApplyMatrix construction tool - Graphics window?	"transform(s) to" - Finding new situations' transformation - Differentiating algebraic and geometric meanings of new situations - "Linear equations" - Making connection: matrix representation and the situation - Comprehending output equations as "linear" or "non-linear"	







Summary: DBR

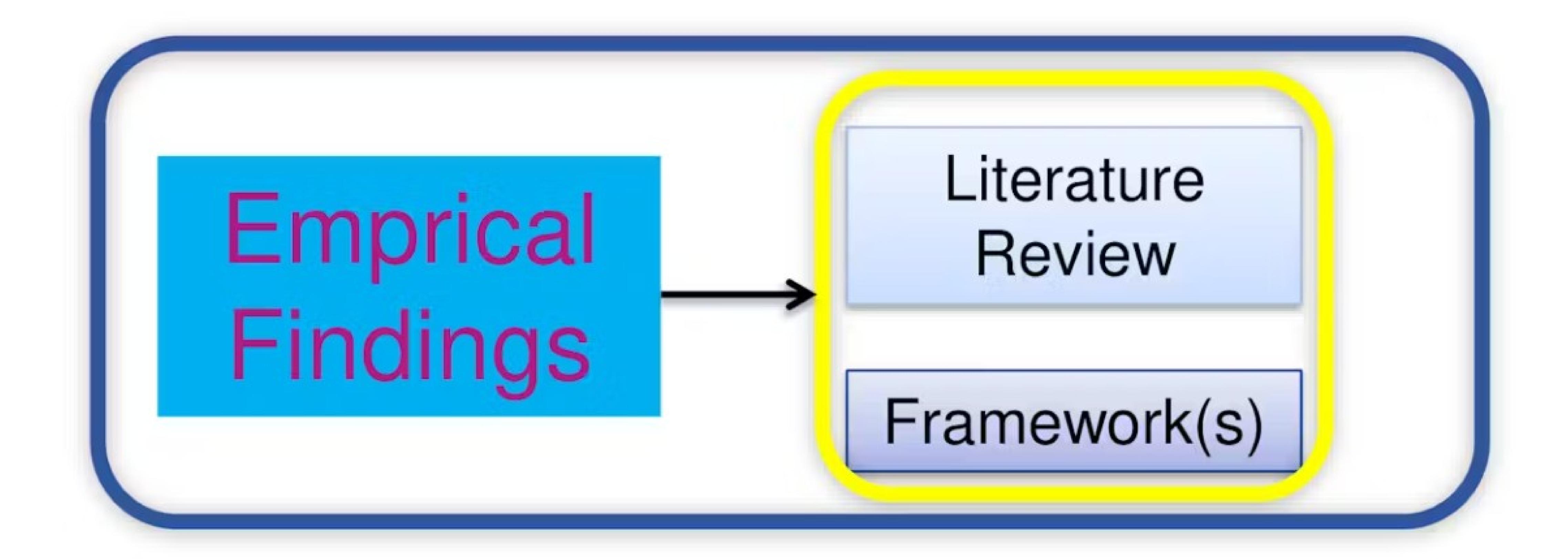




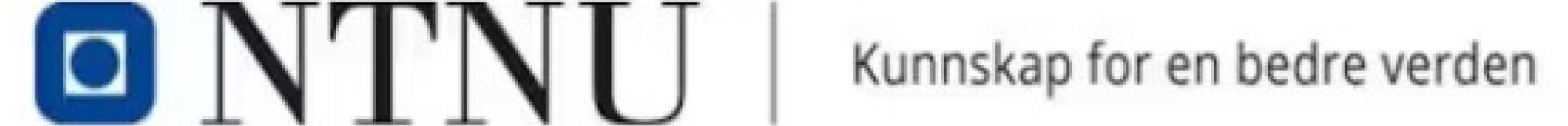


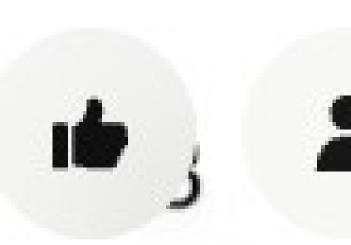


Local Instruction Theories



Example: A framework for students' learning of geometric matrix transformations

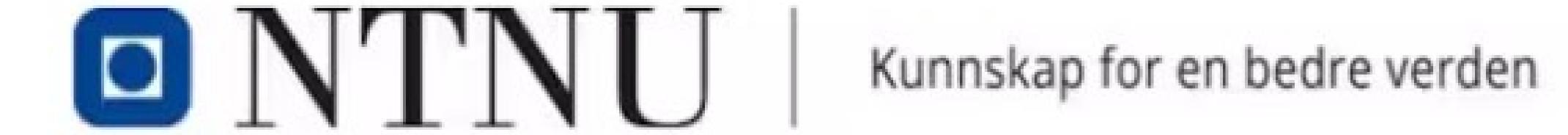






References

- Bakker, A., van Eerde, D. (2015). An Introduction to Design-Based Research with an Example From Statistics Education. In: Bikner-Ahsbahs, A., Knipping, C., Presmeg, N. (eds) Approaches to Qualitative Research in Mathematics Education. Advances in Mathematics Education. Springer, Dordrecht. https://doi.org/10.1007/978-94-017-9181-6 16
- Plomp, T. (2010). Educational Design Research: an Introduction. An Introduction to Educational Design Research. SLO.
- Turgut, M. (2019). Sense-making regarding matrix representation of geometric transformations in: a semiotic mediation perspective in a dynamic geometry environment. ZDM Mathematics Education 51, 1199–1214. https://doi.org/10.1007/s11858-019-01032-0







Please indicate your thinking:)





0 It does not fit to my plans/aim





0 I have many questions



