UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY TWO-YEAR MASTER OF SCIENCE PROGRAMME IN MATHEMATICS

ACADEMIC YEAR 2024-2025

ACADEMIC SUBJECT ELEMENTARY MATHEMATICS FROM AN ADVANCED POINT OF VIEW

General information	
Term	First semester (September 23, 2024 – December 20, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/04 – Complementary Mathematics
Language	Italian
Mode of attendance	Not mandatory

Lecturers		
Name and surname	Roberto Capone (instructor of record)	Eleonora Faggiano
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Department and office	Department of Mathematics room 3 second floor	Department of Mathematics room 4 second floor
Virtual meeting room		
Web page	https://www.dm.uniba.it/it/members/capone	https://www.dm.uniba.it/it/members/faggiano
Office hours		

Work schedule				
	Total	Lectures	Hands-on learning	Self-study
Hours	175	56		119
ECTS credits	7	7		

Learning objectives	
	The course frames elementary mathematics (with content related to geometry, arithmetic, analysis, set theory) from an epistemological and didactic perspective. In particular, the basic tools for preparing teaching activities and discussing these topics with students are provided. This teaching naturally fits into the Educational Curriculum but can be taken by anyone who shows interest in the cultural and educational dimensions of mathematics.

Course prerequisites	
	The background course content of geometry, algebra and mathematical analysis are important

Syllabus	
Course contents	**Rational Arithmetic and Algebra for Teaching** Axioms of Peano arithmetic and development of PA arithmetic. The concept of natural number according to Frege. Natural numbers and cardinality: the

	antinomies of set theory and the Cantor-Bernstein theorem. The concept of natural number according to Dedekind. Inductive definitions and recursion theorem. Husserl's Concept of Natural Number: Numbers and Cognitive Science. Fractions. Analytic and algebraic reasons for Q's incompleteness. Dedekind's construction of the field of real numbers and R's completeness theorems. The axioms of continuity. Real numbers and measurement. Méray-Cantor construction of the field of real numbers and metric completeness theorems. Hilbert's construction of the field of real numbers and foundational problems: coherence, categoricity, independence. Introduction to filters and ultrafilters. Hyperreal field construction and non-standard analysis. Non-Archimedean fields. **Didactics of Elementary Algebra** • the notion of symbol sense (Arcavi) • operational and structural conceptions in mathematics (e.g., Sfard) • The arithmetic-algebra gap • Algebra skills: modeling, proving numerical properties, reasoning **Synthetic Geometry for Teaching** The mathematics of the Egyptians, Sumerians and Babylonians Greek mathematics: the Pythagorean school: all things that are known have a number, the Pythagorean scale, numbers are not enough: incommensurable magnitudes. The Pythagorean theorem is a proof of it; the inverse theorem of the Pythagorean theorem. Another way to prove the Pythagorean theorem (starting with Euclid's first theorem). Surfaces and equivalent surfaces. Euclid's first theorem. Prime numbers. Paradox of the existence of non-commensurable quantities. Theorem on the existence of impossible triangles. Demonstrations and demonstrations by absurdity. Thales. Theorem on the infinity of prime numbers. Euclid and his Elements. Contents of Book I: Definitions, Postulates, Axioms, Constructive Propositions, A Wrong Proof, The Three Criteria of Congruence; Pons Asinorum theorem, theorem (I, 48). The theory of homogeneous quantities, Archimedes' postulate, axiom of continuity, commensurable quantities, proportionality; Theorem of existence of the proportional fourth. Equi-composability. Equivalence between a parallelogram and a rectangle, equivalence between a triangle and a rectangle, Plato and the duplication of the square Euclid and the theory of homogeneous quantities, equicomposability of a regular polygon and a rectangle; equidecomposability as an equivalence relation; first and second theorems of the outer angle; parallelism criterion. Ceva's theorem, from mathematics to statics, Menelaus' theorem, Stewart's theorem, Cevian triangles, nettles, pedals; Fagnano's problem, proof with Fej'er axial symmetries, nettle quadrilaterals; Varignon's theorem, principal hortic quadrilaterals; Billiards: flat billiards and polygonal billiards. **Didactics of Geometry for Teaching** Intuitive Geometry vs Rational Geometry **Analysis for Teaching** The birth of analytic geometry (R. Descartes, P. Fermat). Infinitesimal calculus in Newton's works (method of fluxions, method of the first last ratios, method of series, the "fundamental theorem of integral calculus", integration of functions, integration of differential equations). The infinitesimal calculus in Leibniz. The comparison between the Schools of Leibniz and Newton. The premises for the creation of infinitesimal calculus (Cavalieri, Torricelli, Barrow). The series in the eighteenth century (outline).

	Evolution of the concept of function. La *Théorie des fonctions analytiques* (1797) di J.-L. Lagrange and the algebraization of analysis. Cauchy and the beginning of the process of rigorizing the analysis: il *Cours d'analyse* (1821) e i *Résumés des leçons données à l'Ecole royale polytechnique sur le calcul infinitésimal* (1823). The evolution of the concepts of limit, derivative and integral in the eighteenth and nineteenth centuries. **Didactics of Elementary Analysis:** • history and epistemology of the concept of function: process and object (Sfard); the notion of covariation of variables (Slavit) • The Mathematics of Change (Kaput) • The algebra-analysis gap • the cognitive roots of concepts (Tall)
Reference books	Capone – Faggiano, *MATEMATICHE ELEMENTARI DA UN PUNTO DI VISTA SUPERIORE*, eBook provided to students in pdf Other recommended readings: • Prodi, Analisi matematica, Boringhieri • Gerla, Tentativi di fondare la matematica, voll. 1 e 2 • Boyer, Storia del Calcolo
Additional course materials	
Repository	Teaching materials will be provided to students lesson by lesson following the Just in Time Teaching protocol

Expected learning outcomes	
Knowledge and understanding	The course allows students to acquire a knowledge of the fundamentals of algebra, geometry and analysis and to re-read some topics in an epistemological and didactic key with the aim of 1) improving critical skills also thanks to the use of different sources and texts; 2) improve problem-solving skills; 3) consolidate the mastery of the concepts and the scientific method applied to teaching**.** Students will be accustomed to the use of theoretical and epistemological knowledge to build educational activities for secondary school.
Applying knowledge and understanding	The proposed activities develop students' ability to solve and pose problems, establishing links between various areas of mathematics as well as the ability to use the skills acquired, both for research purposes and to develop teaching activities for secondary schools.
Soft skills	*Making judgements*: The nature of this teaching induces students to improve their critical and argumentative skills and accustoms them to: recognize errors or gaps in proofs, reflect on the change in methodologies and mathematical tools over time, independently develop examples of activities for secondary school, ask themselves didactic and in-depth questions that give cultural value to the topics covered.
	Communication skills: The presentation of a didactic activity and collective discussions accustom students to present their point of view, to argue, to understand the point of view of others, using various communication tools.
	Learning skills: Students acquire the ability to set up work rigorously and solve theoretical problems, to critically approach the study of a mathematical text, to communicate mathematics to a non-mathematical audience through the implementation of educational activities for secondary school.

Teaching methods	
	Interactive Lessons, Just in Time Teaching,

<table>
<tr><th colspan="2">Assessment</th></tr>
<tr><td>Assessment methods</td><td>Group and individual work and collective discussions during the course for the formative assessment. Oral exam at the end of the course for summative assessment</td></tr>
<tr><td>Evaluation criteria</td><td>• Knowledge and understanding: Knowledge of didactic contents
• Applying knowledge and understanding: Ability to use acquired knowledge to solve problems
• Making judgement: Knowledge of the epistemology that characterizes the main concepts and methods presented
• Communication skills: Ability to place mathematics in a broader cultural context and to communicate it
• Learning skills: Ability to use the acquired knowledge to prepare didactic activities for secondary school</td></tr>
<tr><td>Grading policy</td><td>
<table>
<tr><td>A
Advanced</td><td>B
High Level</td><td>C
Medium level</td><td>D
Beginning</td></tr>
<tr><td>The Student has a perfect command of the topics covered in the teaching. Master the tools of calculus, algebra, and geometry without hesitation of procedural execution. Appropriately use discipline-specific language to communicate the content studied. He/she is able to prepare didactic activities, starting from the contents studied, in full autonomy and with originality</td><td>The Student is well acquainted with all the topics covered in the teaching. Use the tools of calculus, algebra, and geometry appropriately. It uses the specific language of the discipline to communicate the contents studied. He/she is able to prepare didactic activities, starting from the contents studied, independently and correctly.</td><td>The Student knows the topics covered in the course. He uses the tools of calculus, algebra and geometry, although sometimes with some hesitation. It uses a correct language of the discipline to communicate the contents studied. He/she is able to prepare didactic activities, starting from the contents studied, sometimes with the support of auxiliary aids</td><td>The Student knows almost all the topics covered in the teaching. Use the tools of calculus, algebra, and geometry with some hesitation. It uses the language of the discipline to communicate the contents studied, even if with some uncertainty about the presentation. He/she is able to prepare didactic activities, starting from the contents studied, only if supported by auxiliary aids.</td></tr>
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<tr><th>FASCIA</th><th>CORRISPONDENZA TASSONOMICA</th><th>LIVELLO DI COMPETENZA</th></tr>
<tr><td>A</td><td>27-30</td><td>ADVANCED</td></tr>
<tr><td>B</td><td>23-26</td><td>High</td></tr>
<tr><td>C</td><td>20-22</td><td>Medium</td></tr>
<tr><td>D</td><td>18 - 19</td><td>beginning</td></tr>
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Further information	