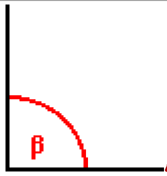
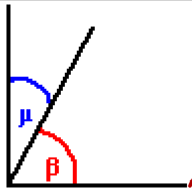
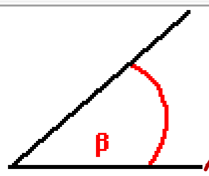
Formulario di trigonometria



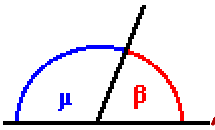
ANGOLO RETTO (β = 90°)



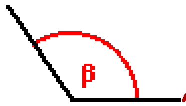
ANGOLI COMPLEMENTARI (β + μ = 90°)



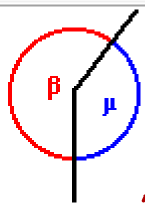
ANGOLO ACUTO (β < 90°)



ANGOLI SUPPLEMENTARI (β + μ = 180°)



ANGOLO OTTUSO (β > 90°)



ANGOLI ESPLEMENTARI (β + μ = 360°)

ANGOLI COMPLEMENTARI	
• $\sin(90° - \beta) = \cos\beta$ • $\cos(90° - \beta) = \sin\beta$ • $tg(90° - \beta) = ctg\beta$ • $ctg(90° - \beta) = tg\beta$	• $\sin(\pi/2 - \beta) = \cos\beta$ • $\cos(\pi/2 - \beta) = \sin\beta$ • $tg(\pi/2 - \beta) = ctg\beta$ • $ctg(\pi/2 - \beta) = tg\beta$
ANGOLI CHE DIFFERISCONO DI UN ANGOLO RETTO	
• $\sin(90° + \beta) = \cos\beta$ • $\cos(90° + \beta) = -\sin\beta$ • $tg(90° + \beta) = -ctg\beta$ • $ctg(90° + \beta) = -tg\beta$	• $\sin(\pi/2 + \beta) = \cos\beta$ • $\cos(\pi/2 + \beta) = -\sin\beta$ • $tg(\pi/2 + \beta) = -ctg\beta$ • $ctg(\pi/2 + \beta) = -tg\beta$

ANGOLI SUPPLEMENTARI	
• $\sin(180° - \beta) = \sin\beta$ • $\cos(180° - \beta) = -\cos\beta$ • $tg(180° - \beta) = -tg\beta$ • $ctg(180° - \beta) = -ctg\beta$	• $\sin(\pi - \beta) = \sin\beta$ • $\cos(\pi - \beta) = -\cos\beta$ • $tg(\pi - \beta) = -tg\beta$ • $ctg(\pi - \beta) = -ctg\beta$
ANGOLI CHE DIFFERISCONO DI UN ANGOLO PIATTO	
• $\sin(180° + \beta) = -\sin\beta$ • $\cos(180° + \beta) = -\cos\beta$ • $tg(180° + \beta) = tg\beta$ • $ctg(180° + \beta) = ctg\beta$	• $\sin(\pi + \beta) = -\sin\beta$ • $\cos(\pi + \beta) = -\cos\beta$ • $tg(\pi + \beta) = tg\beta$ • $ctg(\pi + \beta) = ctg\beta$

ANGOLI CHE HANNO PER SOMMA TRE ANGOLI RETTI	
• $\sin(270° - \beta) = -\cos\beta$ • $\cos(270° - \beta) = -\sin\beta$ • $tg(270° - \beta) = ctg\beta$ • $ctg(270° - \beta) = tg\beta$	• $\sin(\frac{3\pi}{2} - \beta) = -\cos\beta$ • $\cos(\frac{3\pi}{2} - \beta) = -\sin\beta$ • $tg(\frac{3\pi}{2} - \beta) = ctg\beta$ • $ctg(\frac{3\pi}{2} - \beta) = tg\beta$
ANGOLI CHE DIFFERISCONO DI TRE ANGOLI RETTI	
• $\sin(270° + \beta) = -\cos\beta$ • $\cos(270° + \beta) = \sin\beta$ • $tg(270° + \beta) = -ctg\beta$ • $ctg(270° + \beta) = -tg\beta$	• $\sin(\frac{3\pi}{2} + \beta) = -\cos\beta$ • $\cos(\frac{3\pi}{2} + \beta) = \sin\beta$ • $tg(\frac{3\pi}{2} + \beta) = -ctg\beta$ • $ctg(\frac{3\pi}{2} + \beta) = -tg\beta$

ANGOLI ESPLEMENTARI	
• $\sin(360° - \beta) = -\sin\beta$ • $\cos(360° - \beta) = \cos\beta$ • $tg(360° - \beta) = -tg\beta$ • $ctg(360° - \beta) = -ctg\beta$	• $\sin(2\pi - \beta) = -\sin\beta$ • $\cos(2\pi - \beta) = \cos\beta$ • $tg(2\pi - \beta) = -tg\beta$ • $ctg(2\pi - \beta) = -ctg\beta$

ANGOLI OPPOSTI

- $\sin(-\beta) = -\sin\beta$
- $\cos(-\beta) = \cos\beta$
- $tg(-\beta) = -tg\beta$
- $ctg(-\beta) = -ctg\beta$

VALORI				
noto	$\sin\beta$	$\cos\beta$	$tg\beta$	$ctg\beta$
$\sin\beta$	$\sin\beta$	$\pm\sqrt{1-\sin^2\beta}$	$\pm\frac{\sin\beta}{\sqrt{1-\sin^2\beta}}$	$\pm\frac{\sqrt{1-\sin^2\beta}}{\sin\beta}$
$\cos\beta$	$\pm\sqrt{1-\cos^2\beta}$	$\cos\beta$	$\pm\frac{\sqrt{1-\cos^2\beta}}{\cos\beta}$	$\pm\frac{\cos\beta}{\sqrt{1-\cos^2\beta}}$
$tg\beta$	$\frac{tg\beta}{\pm\sqrt{1+tg^2\beta}}$	$\frac{1}{\pm\sqrt{1+tg^2\beta}}$	$tg\beta$	$\frac{1}{tg\beta}$
$ctg\beta$	$\pm\frac{1}{\sqrt{1+ctg^2\beta}}$	$\pm\frac{ctg\beta}{\sqrt{1+ctg^2\beta}}$	$\frac{1}{ctg\beta}$	$ctg\beta$

GRADI	RADIANTI	SENO	COSENO	TANGENTE	COTANGENTE
0°	0	0	1	0	non esiste
15°	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18°	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{5+2\sqrt{5}}$
22°30'	$\frac{\pi}{8}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
36°	$\frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
75°	$\frac{5}{12}\pi$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	non esiste	0

Formule di sottrazione	Formule di addizione
$\sin(\mu-\beta) = \sin\mu\cos\beta - \cos\mu\sin\beta$	$\sin(\mu+\beta) = \sin\mu\cos\beta + \cos\mu\sin\beta$
$\cos(\mu-\beta) = \cos\mu\cos\beta + \sin\mu\sin\beta$	$\cos(\mu+\beta) = \cos\mu\cos\beta - \sin\mu\sin\beta$
$tg(\mu-\beta)=(tg\mu - tg\beta)/(1 + tg\mu tg\beta)$	$tg(\mu+\beta) = (tg\mu + tg\beta)/(1 - tg\mu tg\beta)$
$ctg(\mu-\beta) = (ctg\mu ctg\beta + 1)/(ctg\beta - ctg\mu)$	$ctg(\mu+\beta) = (ctg\mu ctg\beta - 1)/(ctg\beta + ctg\mu)$

Formule di duplicazione
$\sin 2\beta = 2 \sin\beta \cos\beta$
$\cos 2\beta = \cos^2\beta - \sin^2\beta = 1 - 2\sin^2\beta = 2\cos^2\beta - 1$
$tg2\beta = \frac{2tg\beta}{1 - tg^2\beta}$
$ctg2\beta = \frac{ctg^2\beta - 1}{2ctg\beta}$

Formule di bisezione
$\sin\frac{\beta}{2} = \pm\sqrt{\frac{1-\cos\beta}{2}}$
$\cos\frac{\beta}{2} = \pm\sqrt{\frac{1+\cos\beta}{2}}$
$tg\frac{\beta}{2} = \pm\sqrt{\frac{1-\cos\beta}{1+\cos\beta}} = \frac{\sin\beta}{1+\cos\beta} = \frac{1-\cos\beta}{\sin\beta}$
$ctg\frac{\beta}{2} = \pm\sqrt{\frac{1+\cos\beta}{1-\cos\beta}} = \frac{1+\cos\beta}{\sin\beta} = \frac{\sin\beta}{1-\cos\beta}$

Formule di prostaferesi
$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$

$\sin p - \sin q = 2\sin\frac{p-q}{2}\cos\frac{p+q}{2}$
$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$
$\cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$
$\text{tg}p \pm \text{tg}q = \frac{\sin(p \pm q)}{\cos p \cos q}$ [con p e q $\neq (2k+1)\pi/2$]
$\text{ctg}p \pm \text{ctg}q = \frac{\sin(q \pm p)}{\sin p \sin q}$ [con p e q $\neq k\pi$]

Formule di Werner
$\sin\mu\cos\beta = \frac{1}{2}[\sin(\mu+\beta) + \sin(\mu-\beta)]$
$\cos\mu\cos\beta = \frac{1}{2}[\cos(\mu+\beta) + \cos(\mu-\beta)]$
$\sin\mu\sin\beta = \frac{1}{2}[\cos(\mu-\beta) - \cos(\mu+\beta)]$

Espressione di sinβ, cosβ, tgβ, ctgβ, in funzione razionale di tg(β/2)	
$\sin\beta = \frac{2\text{tg}\frac{\beta}{2}}{1+\text{tg}^2\frac{\beta}{2}}$	$\cos\beta = \frac{1-\text{tg}^2\frac{\beta}{2}}{1+\text{tg}^2\frac{\beta}{2}}$
$\text{tg}\beta = \frac{2\text{tg}\frac{\beta}{2}}{1-\text{tg}^2\frac{\beta}{2}}$	$\text{ctg}\beta = \frac{1-\text{tg}^2\frac{\beta}{2}}{2\text{tg}\frac{\beta}{2}}$

TRIGONOMETRIA

Relazioni tra gli elementi di un triangolo rettangolo	
$b = a \sin\beta$	$c = b \cos\beta$
$b = c \,tg\beta$	$c = b \,ctg\beta$

Teorema della corda
$AB = 2r \sin\beta$

Teorema dei seni (o di Eulero)
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

Teorema delle proiezioni
$a = b \cos\gamma + c \cos\beta$
$b = a \cos\gamma + c \cos\alpha$
$c = a \cos\beta + b \cos\alpha$

Teorema del coseno (o di Carnot)
$a^2 = b^2 + c^2 - 2\,bc \cos\alpha$ $b^2 = a^2 + c^2 - 2\,ac \cos\beta$ $c^2 = a^2 + b^2 - 2\,ab \cos\gamma$

Applicazioni geometriche della trigonometria	
Calcolo dell'area di un triangolo	$A = \frac{ab \sin \gamma}{2}$
	$A = \frac{a^2 \sin \beta \sin \gamma}{2 \sin \alpha}$
	$A = \sqrt{p(p-a)(p-b)(p-c)}$
Calcolo dell'area di un quadrilatero	$A = \frac{dd' \sin \alpha}{2}$
Raggio delle circonferenze che, rispetto ad un triangolo qualsiasi, sono	inscritte $r = \frac{A}{p} = (p-a) \operatorname{tg} \frac{\alpha}{2}$
	circoscritte $r = \frac{abc}{4A}$
	exinscritte $r_1 = \frac{A}{p-a}$ $r_2 = \frac{A}{p-b}$ $r_3 = \frac{A}{p-c}$
Mediane di un triangolo	$m_a = \frac{\sqrt{2b^2 + 2c^2 - a^2}}{2}$
	$m_b = \frac{\sqrt{2a^2 + 2c^2 - b^2}}{2}$
	$m_c = \frac{\sqrt{2a^2 + 2b^2 - c^2}}{2}$
Bisettrici di un triangolo	$b_\alpha = \frac{2bc \cos \frac{\alpha}{2}}{b + c}$
	$b_\beta = \frac{2ac \cos \frac{\beta}{2}}{a + c}$
	$b_\gamma = \frac{2ab \cos \frac{\gamma}{2}}{a + b}$

Teorema di Nepero

$$\frac{a-b}{a+b}=\frac{\operatorname{tg}\frac{\alpha-\beta}{2}}{\operatorname{tg}\frac{\alpha+\beta}{2}}$$

Formule di Briggs

$\sin\frac{\alpha}{2}=\sqrt{\frac{(p-b)(p-c)}{bc}}$	$\cos\frac{\alpha}{2}=\sqrt{\frac{p(p-a)}{bc}}$
$\sin\frac{\beta}{2}=\sqrt{\frac{(p-a)(p-c)}{ac}}$	$\cos\frac{\beta}{2}=\sqrt{\frac{p(p-b)}{ac}}$
$\sin\frac{\gamma}{2}=\sqrt{\frac{(p-a)(p-b)}{ab}}$	$\cos\frac{\gamma}{2}=\sqrt{\frac{p(p-c)}{ab}}$
$\operatorname{tg}\frac{\alpha}{2}=\sqrt{\frac{(p-b)(p-c)}{p(p-a)}}$	$\operatorname{ctg}\frac{\alpha}{2}=\sqrt{\frac{p(p-a)}{(p-b)(p-c)}}$
$\operatorname{tg}\frac{\beta}{2}=\sqrt{\frac{(p-a)(p-c)}{p(p-b)}}$	$\operatorname{ctg}\frac{\beta}{2}=\sqrt{\frac{p(p-b)}{(p-a)(p-c)}}$
$\operatorname{tg}\frac{\gamma}{2}=\sqrt{\frac{(p-a)(p-b)}{p(p-c)}}$	$\operatorname{ctg}\frac{\gamma}{2}=\sqrt{\frac{p(p-c)}{(p-a)(p-b)}}$